A Precise and Practical Harmonic Elimination Method for Multilevel Inverters

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***Abstract*—Multilevel inverters have been widely used in medium- and high-voltage applications. Selective harmonic elimination for the staircase voltage waveform generated by multilevel inverters has been studied extensively in the last decade. Most of the published methods on this topic were based on solving high-order multivariable polynomial equation groups derived from Fourier series expansion. This paper presents a different approach, which is based on equal area criteria and harmonic injection. With the proposed method, regardless of how many voltage levels are involved, only four simple equations are needed. The results of a case study with maximum of five switching angles show that the proposed method can be used to achieve excellent harmonic elimination performance for the modulation index range at least from 0.2 to 0.9. To demonstrate the adaptability of the proposed method for waveforms with a high number of switching angles, experimental results on a 1-MVA 6000-V 17-level cascade multilevel inverter are also shown at the end of this paper.**

***Index Terms*—Equal area criteria, modulation index, multilevel inverters, pulsewidth modulation (PWM), total harmonic distortion (THD).**

I. Introduction

IN MEDIUM- and high-voltage applications, the implementation of high-frequency pulsewidth-modulation (PWM)-based two-level inverters is limited due to voltage and current ratings of switching devices, switching losses, and electromagnetic interferences caused by high dv/dt. Thus, to overcome these limitations, multilevel inverters have been proposed for applications such as medium-voltage drives, renewable energy interfaces, and flexible ac transmission devices [1]–[11]. A typical multilevel inverter utilizes voltage levels from multiple dc sources. These dc sources can be isolated as in cascade multilevel structures or interconnected as in diode-clamped structures. In most published multilevel inverter circuit topologies, the dc sources in the circuits need to be maintained to supply identical voltage levels. Based on these identical voltage levels and proper control of the switching angles of the switches, a staircase waveform can be synthesized, such as a six-level staircase waveform with five switching angles shown in Fig. 1.

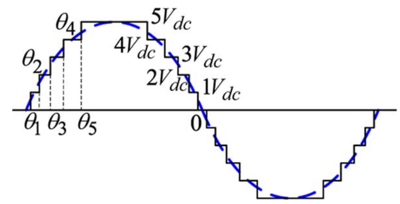

Fig. 1. General staircase waveform of multilevel inverters.

One of the greatest benefits of this staircase waveform is that the switches in the inverters only need to be switched on and off once during one fundamental cycle; thus, the switching loss of the devices is reduced to minimum. However, with reduced switching frequencies, even with additional voltage levels, low-frequency harmonics can be found in this type of staircase voltage [12]–[20], [28], [29].

II. Existing Harmonic Elimination Methods

Until now, there are two major approaches to eliminate low-frequency harmonics: 1) increasing the switching frequency in sinusoidal–triangular PWM and space vector PWM for two-level inverters or adopting phase shift in multicarrier-based PWM for multilevel inverters [21]–[24] and 2) optimizing switching angles for selected harmonic elimination (SHE) [12]–[20]. The first approach is limited by switching loss and is usually used when the available voltage steps are limited, e.g., two or three steps.

SHE-based methods have been proposed for both two-level [26]–[28] and multilevel inverters. This paper is focusing on the SHE-based methods for multilevel inverters. Ideally, in the multilevel inverters, for every voltage level, there could be multiple switching angles. The number of eliminated harmonics is decided by the number of voltage steps and number of switching angles in each voltage step. However, because of the complexity of the problem, most studies proposed so far are for one switching angle per one voltage level, as shown in Fig. 1. In this case, the Fourier series expansion of the staircase waveform can be expressed as

$$V(\omega t)=\sum_{m=1,3,5,\ldots}^{\infty}\frac{4V_{\rm dc}}{m\pi}(\cos(m\theta_1)+\cdots\cos(m\theta_N))\sin(m\omega t) \quad (1)$$

where N is the number of switching angels and m is the harmonic order. Based on (1), traditionally, the following

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polynomial equation group can be formed to calculate the switching angles to realize SHE for the multilevel inverter:

$$\begin{cases} \frac{4V_{\rm dc}}{\pi}\left(\cos(\theta_1)+\cos(\theta_2)+\cos(\theta_3)+\cos(\theta_4)+\cos(\theta_5)\right)=V_F \\ \cos(5\theta_1)+\cos(5\theta_2)+\cos(5\theta_3)+\cos(5\theta_4)+\cos(5\theta_5)=0 \\ \cos(7\theta_1)+\cos(7\theta_2)+\cos(7\theta_3)+\cos(7\theta_4)+\cos(7\theta_5)=0 \\ \cos(11\theta_1)+\cos(11\theta_2)+\cos(11\theta_3) \\ \quad +\cos(11\theta_4)+\cos(11\theta_5)=0 \\ \cos(13\theta_1)+\cos(13\theta_2)+\cos(13\theta_3) \\ \quad +\cos(13\theta_4)+\cos(13\theta_5)=0. \end{cases} \tag{2}$$

In this equation group, the first equation guarantees the desired fundamental component V_F. The second to the fifth equations are utilized to ensure the elimination of 5th, 7th, 11th, and 13th harmonics. It is clear that, with five switching angles, four selected harmonics can be eliminated. The SHE methods proposed in [12]–[20] essentially are methods that try to solve the equation group (2) with different approaches. Ideally, by solving these polynomial equations, the selected harmonic components can be eliminated very precisely.

However, due to the nature of high-order polynomial equation groups, there are also several disadvantages of these kinds of methods. One of the main difficulties of applying most of these methods in real engineering practice is that, when the number of dc levels increases, the number of polynomial equations, the number of variables, and the order of the equations will all increase accordingly. Thus, finding solutions to these equations would become extremely difficult and often involve advanced mathematical algorithms, which make the calculation easy to reach the capability limits of existing computer algebra software tools [15]. Although advanced methods such as the symmetric polynomials and resultant theory combined method [15] and generic-algorithm-based methods [16], [19] can greatly reduce the calculation time, these methods are difficult to be adopted by field engineers because of the need for preunderstanding of advanced control and mathematic theories. A more detailed summary of the different types of SHE method can also be found in a very recently published paper [20]. In short, although many methods have been proposed to solve the SHE problem in multilevel inverters, a simple and practical method is still needed.

In this paper, the concept of a four-simple-equation-based method is first introduced. Then, the problems of the direct implementation of the proposed method are analyzed. Solutions are proposed accordingly to enable the good performance of the method at a wide range of modulation index. At the end of this paper, full calculation results of switching angles for an inverter with maximum of five switching angels show that the proposed method can be used for the modulation index range covered by all the other methods proposed so far. To show the effectiveness of the proposed method in applications with large numbers of switching angles, experimental results on a 1-MVA 6000-V 17-level cascade multilevel inverter are also shown at the end of this paper.

Aside from offline calculation methods addressed in this paper, real-time-based SHE methods and algorithms for unbalanced dc sources have also been often discussed [18], [26], [29].

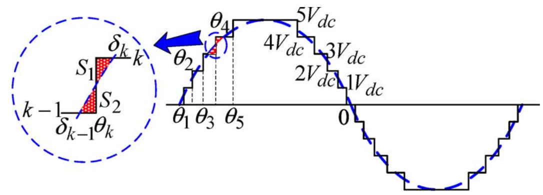

Fig. 2. Diagram showing the idea for equal area criteria.

Very recently, a simple method that avoids polynomial equations in switching angle calculation is proposed in [29] for real-time calculation, where a second-order equation has been proposed to minimize the total harmonic distortion (THD) of the inverter output voltage. However, because of the simplification, the THD in output voltage is much higher than that of the methods using polynomial equations. Thus, since more research is still needed, the future development direction of the method proposed in this paper will be online switching angle calculations.

III. Four-Equation-Based Harmonic Elimination Method

A. Principle of the Proposed Method

The proposed method in this paper tries to solve the harmonic elimination problems from a totally different approach. No high-order multivariable polynomial equations would be involved in this method. To better illustrate the proposed method, two well-known examples of switching angle calculation and harmonic compensation are first introduced as follows.

1) Equal Area Criteria for Switching Angle Calculation: For a simple equation group as (2), the Newton–Raphson iteration can be used to achieve numerical solutions. For Newton–Raphson-based iteration, the initial values are very crucial to the final results. One natural way to find good initial switching angles is through equal area criterion. The basic idea of equal area criteria is shown in the circled area in Fig. 2. The initial switching angle θ_k can be found by solving

$$S_1 = S_2 \tag{3}$$

where S_1 and S_2 are the areas of the shadowed parts. By the nature of the equal area criteria, the fundamental of the staircase waveform resulted from the switching angles would resemble the sinusoidal modulation waveform. However, with equal area criteria alone, no harmonic elimination can be realized. How to utilize the initial values from the equal area criteria to find the optimized angles without solving the high-order multivariable polynomials is the question that the method proposed in this paper tries to answer. However, before reaching the final answer, a harmonic elimination method used in utility application is first introduced as follows.

2) Harmonic Injection in APFs: In the power distribution system, active power filters (APFs) are used to eliminate voltage/current harmonics in utility power lines. To eliminate harmonics that are already existing in the utility power lines, APF will inject new harmonic voltages or currents to the lines. The injected harmonics would have the same amplitudes but opposite phase angles of the aimed harmonics. Thus, the harmonics in the utility line could be canceled. The key idea of APF is count-harmonic injection. By combining the equal area criteria and the idea of harmonic injection together, a new method to find optimum switching angles can be found.

B. Proposed Method

The proposed method, indeed, is a combination of equal area criteria and harmonic injection in the modulation waveform. The basic idea behind this method is described as follows [28].

1) By using the equal area criteria, a pure sinusoidal modulation waveform $h_1 = v \sin \omega t$ will result in a set of switching angles $\theta_1 - \theta_N$.
2) The staircase waveform formed by $\theta_1 - \theta_N$ will have the fundamental component h'_1, and harmonic content $h_3, h_5, h_7, \ldots, h_m$, the fundamental component h'_1, will resemble the sinusoidal modulation waveform h_1.
3) If $h_1 - h_5 - h_7 \cdots - h_m$ is taken as the modulation waveform, by using the equal area criteria, the selected harmonic content in the resulted staircase waveform would be around $h_5 + h_7 \cdots + h_m - h'_5 - h'_7 \cdots - h'_m$, where $h_5 + h_7 \cdots + h_m$ is generated by h_1 and $-h'_5 - h'_7 \cdots - h'_m$ is generated by $-h_5 - h_7 \cdots - h_m$. Again, because of the nature of the equal area criteria, $-h'_5 - h'_7 \cdots - h'_m$ would follow $-h_5 - h_7 \cdots - h_m$ very closely, and the harmonic elimination is partially realized.
4) If the same process in 2)–3) is repeated, harmonic elimination can finally be realized.

To implement this idea, the following five steps need to be followed.

1) First, based on the equal area criteria, find the initial switching angles $(\theta_1 - \theta_N)$ for a given modulation waveform h_1 at a certain modulation index.
2) Then, find the non-third harmonic content $(h_5, h_7, \ldots, h_m)$ of the staircase waveform formed with switching angles $\theta_1 - \theta_N$.
3) Subtract the harmonic content $h_5, h_7, \ldots h_m$ from the original modulation waveform h to form a new modulation waveform $h - h_5 - h_7 \cdots - h_m$; for the first iteration, $h = h_1$.
4) Based on the equal area criteria, use the new nonsinusoidal modulation waveform to calculate a new set of $(\theta_1 - \theta_N)$.
5) Repeat steps 2)–4) until the best switching angles are achieved, which would result in full elimination of selected harmonic content.

In step 1), the modulation waveform is pure sinusoidal. After step 3), the harmonics are already injected; the modulation waveform would never be sinusoidal again. The more iteration causes the more harmonics in the modulation waveform. Although the final modulation waveform has large injected harmonic content, the staircase waveform formed by the final switching angles would have almost no selected harmonics.

C. Four Equations

To perform the five steps listed earlier, there are only four equations that need to be calculated.

1) To use the equal area criteria, δ_k, which is the junction point of the modulation waveform and voltage level k, must first be found. For a modulation waveform with harmonic contents, it is difficult to find a symbolic solution for δ_k, but a numeric value can easily be found by doing simple Newton–Raphson-based iterations of the following equation:

$$\delta_k = \arctan\left(\frac{k \cdot V_{\mathrm{dc}} + h_5 \sin(5\delta_k) \cdots h_m \sin(m\delta_k)}{V_F \cos(\delta_k)}\right). \quad (4)$$

2) After δ_k's are found, the switching angle θ_k can easily be calculated from

$$\begin{aligned}\theta_k = {} & k\delta_k - (k-1)\delta_{k-1} \\ & + V_F\left(\cos(\delta_k) - \cos(\delta_{k-1})\right) \\ & - \frac{h_5}{5}\left(\cos(5\delta_k) - \cos(5\delta_{k-1})\right) \\ & - \cdots - \frac{h_m}{m}\left(\cos(m\delta_k) - \cos(m\delta_{k-1})\right) \quad (5)\end{aligned}$$

where m is the order of the harmonic.

3) With a new set of θ_k's, the new harmonic contents can be found as

$$h_m = \sum_{k=1,2,\ldots,N}^{N} \frac{2}{(2k-1)\pi}\left(\cos(m\theta_k) - \cos\left(m(\pi - \theta_k)\right)\right). \quad (6)$$

4) To perform iterations of steps 2)–4) mentioned in this section, the modulation waveform would have a general expression as

$$V_F \sin(\omega t) - h_{5\_s} \sin(5\omega t) - \cdots - h_{m\_s} \sin(m\omega t) \quad (7)$$

where h_{m_s} is the sum of h_m's found after each iteration

$$h_{m\_s} = \sum_{i=1,2,3,\ldots,\mathrm{iter}}^{\mathrm{iter}} h_m. \quad (8)$$

For different numbers of switching angles, the four equations will remain the same. Since no multivariable polynomial equation is involved in this method, the calculation time has a near-linear relationship with the number of switching angles. No sudden increase in calculation time is expected when there is a small change in the number of switching angles. The general diagram of the four-equation-based method is shown in Fig. 3.

D. Problems With the Direct Implementation of the Basic Method

Initially, to prove the concept, this method has been used to calculate the switching angles for the case shown in Fig. 2. In the calculations, five harmonics are chosen for elimination.

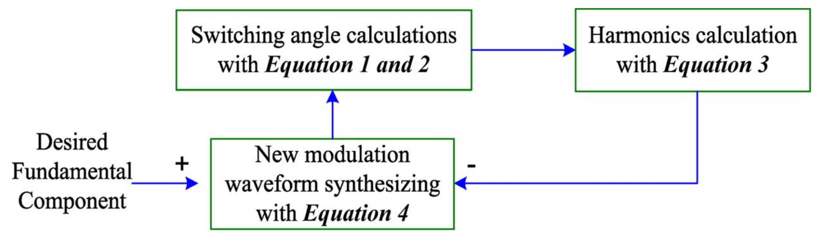

Fig. 3. Four-equation-based method.

TABLE I
SWITCHING ANGLE EXAMPLES

Angles (rad)	θ_1	θ_2	θ_3	θ_4	θ_5
MI=0.85	0.11466	0.25769	0.41205	0.6465	1.0134
MI=0.86	0.11465	0.2577	0.41202	0.64646	1.0134

After about 100 times of iterations, the values of 5th, 7th, 11th, 13th, and 17th harmonics drop under 10^{-6} p.u., which means that these harmonics are effectively eliminated. The number of eliminated harmonics is equal to the number of switching angles N. Please note that, with other methods proposed so far, to eliminate five harmonics with a total of five switching angles would result in fundamental component that is far from the desired value. With the proposed method, because the equal area criteria are used all the time, the resulted fundamental component is still close to the desired value. For a desktop computer with a 2.8-GHz CPU, the calculation time of one modulation index is less than 1 s. Table I shows the switching angle examples for modulation indices at 0.85 and 0.86. The modulation index is defined as

$$MI = \frac{V_F}{\frac{4}{\pi} N \cdot V_{\text{dc}}} \tag{9}$$

where V_F is the peak value of the fundamental component. To identify possible problems with the basic four-equation-based method, the method was tested with five switching angles with modulation index sweeping from 0.16 to 0.94. The main problem identified from this process is the amplitude difference between the desired and resulted fundamental voltages. With the direct implementation of the proposed method, the fundamental voltage of the staircase waveform often diverts from the desired value. The reason is that, for most cases, it is difficult to find a good solution for the switching angle for the top dc level to satisfy the equal area criteria. Table II shows some sample points of the switching angles, harmonic eliminations, and the difference between the desired and resulted modulation indices. It was observed that, when sweeping modulation index from 0.16 to 0.94, for the five-switching-angle-based waveform, the change of the resulted modulation index is not continuous but more like a staircase. Multiple solutions of δ_k for one dc level were originally expected to be another major problem of the four-equation-based method. However, studies show that the equal area criteria can automatically settle on the middle cross point, which is the best selection of δ_k.

IV. Solutions to Problems in This Method

A. PI-Controller-Based Fundamental Voltage Correction

To solve the aforementioned problem, the first attempt was to add a simple proportional-integral (PI) controller to adjust the fundamental component. With this approach, the modulation waveform of this modified method can be expressed as

$$V_{\text{ref}} = (V_F - h_{1s}) * \left(K_p + \frac{K_I}{S}\right) - h_{ms} \sin(m\omega t). \tag{10}$$

The overall diagram of the modified method is shown in Fig. 4. The added process is shown in dotted line. However, with the PI controller in place, there is still a slight difference between the resulted and desired modulation indices for most cases. Since the PI controller is only used for fundamental compensation, the performance of harmonic elimination went bad particularly at high modulation index points. This can be seen in the results listed in Table III. Thus, alternative solutions were proposed and validated as follows.

B. Final Solutions for the Problems in the Four-Equation Method

In the final solutions, the PI controller is no longer used in the iterations. Instead, either an additional voltage level or an additional adjustment of the switching angle at the highest voltage level is used depending on whether an extra voltage level is available at the defined modulation index.

1) Harmonic Elimination With Extra Voltage Level: In multilevel inverters, when the desired modulation index becomes smaller, fewer dc levels will be used to synthesize the staircase waveform. In this case, with the four-equation method, at the fundamental frequency, the difference between the desired voltage and generated voltage will become larger. However, since an extra voltage level is available, it can be utilized to realize the fundamental voltage compensation. Based on this idea, the same five steps in the four-equation method would be used. The difference is that an extra switching angle would be calculated for an extra voltage level to achieve the desired fundamental voltage.

However, the extra voltage level will cause additional harmonics. Thus, in this method, the additional harmonic content generated by the extra voltage level would be added to the overall reference waveform, which is used to calculate the switching angles for all the other voltage levels. This means that the extra harmonics generated in the additional voltage level would be compensated by the switching angles for all the other

TABLE II
SAMPLE POINTS BASED ON THE BASIC METHOD

Reference MI	Resulted MI	Switching Angles (rad.) θ_1	θ_2	θ_3	θ_4	θ_5	Harmonics (%) 5^{th}	7^{th}	11^{th}	13^{th}	17^{th}
0.92	0.8408	0.1147	0.2577	0.4121	0.6465	1.0134	0	0	0	0	0. 584
0.88	0.7923	0.1433	0.3398	0.5275	0.8417	1.1057	0	0	0	0	0.4239
0.84	0.7818	0.1434	0.3406	0.5283	0.8433	1.1062	0	0	0	0	0.5128
0.80	0.7715	0.1435	0.3411	0.5289	0.8443	1.1065	0	0	0	0	0.7062
0.76	0.7251	0.0815	0.4256	0.6818	0.8583	N/A	0	0	0	0.4847	N/A

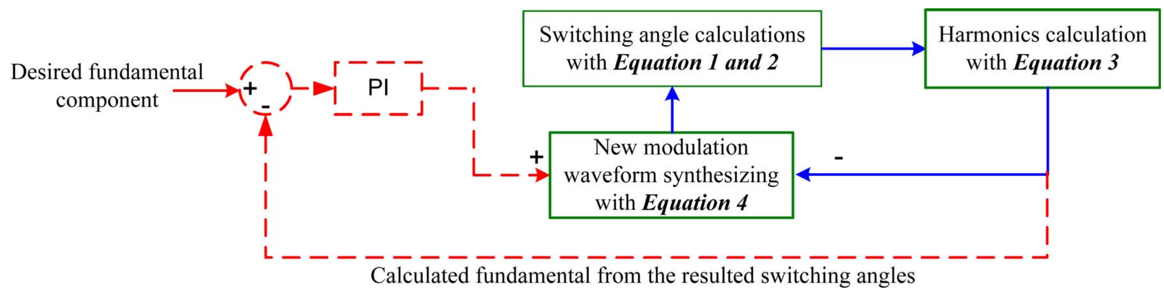

Fig. 4. Modified method with PI controller to adjust the fundamental value.

TABLE III
SAMPLE POINTS FOR MODIFIED METHOD WITH PI CONTROLLER

Reference MI	Resulted MI	Switching Angles (rad.) θ_1	θ_2	θ_3	θ_4	θ_5	Harmonics (%) 5^{th}	7^{th}	11^{th}	13^{th}	17^{th}
0.92	0.9112	0.1794	0.2567	0.315	0.4993	0.7304	0.3976	0.285	0.0085	0.613	0.4304
0.88	0.8467	0.1043	0.2647	0.394	0.6277	0.9994	0.0179	0.0554	0.25	0.4643	0.1339
0.84	0.8308	0.1146	0.2577	0.4119	0.6464	1.0133	0.0008	0.0019	0.0019	0.0038	0.0017
0.80	0.7931	0.1399	0.3182	0.5149	0.802	1.0932	0.3026	0.1336	0.1316	0.7332	0.6585
0.76	0.7351	0.0005	0.2402	0.4088	0.6905	N/A	0.0062	0.0723	0.5558	0.7025	N/A

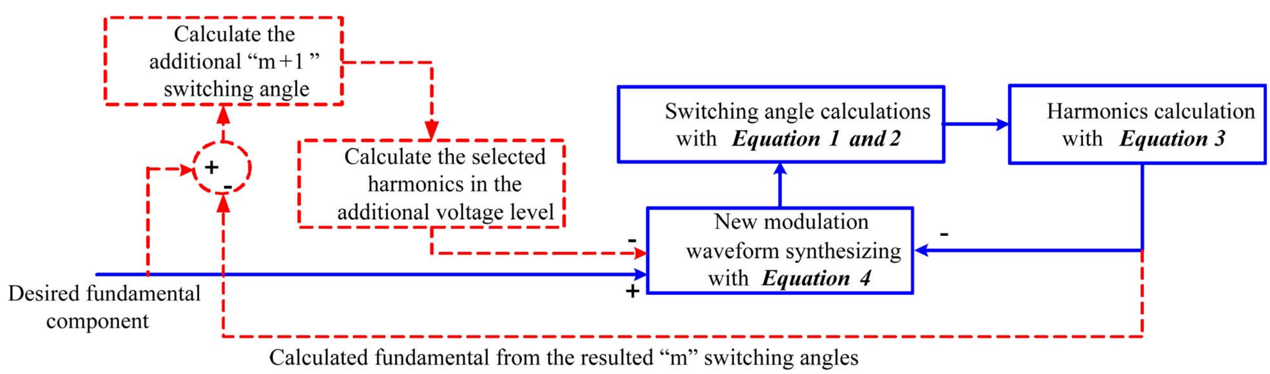

Fig. 5. Modified method with additional switching angle.

voltage levels. This modified method is shown in Fig. 5. The additional process is shown in dotted line.

The following is the detailed procedure on the calculation of the additional "$m+1$" switching angle.

1) First, the total fundamental voltage based on switching angles from θ_1 to θ_m is calculated with the following equation:

$$V_{1m} = \sum_{i=1}^{m} \frac{4V_{\rm dc}}{\pi} \cos(\theta_i), \qquad m < N. \tag{11}$$

2) Then, the switching angle of the additional voltage level is calculated based on the difference between the desired fundamental voltage V_F and the resulted fundamental voltage V_{1m}

$$\theta_{m+1} = a\cos\left(\frac{\pi}{4V_{\rm dc}}(V_F - V_{1m})\right). \tag{12}$$

2) Harmonic Elimination With no Extra Voltage Levels: For larger modulation indices, where all dc levels are already used

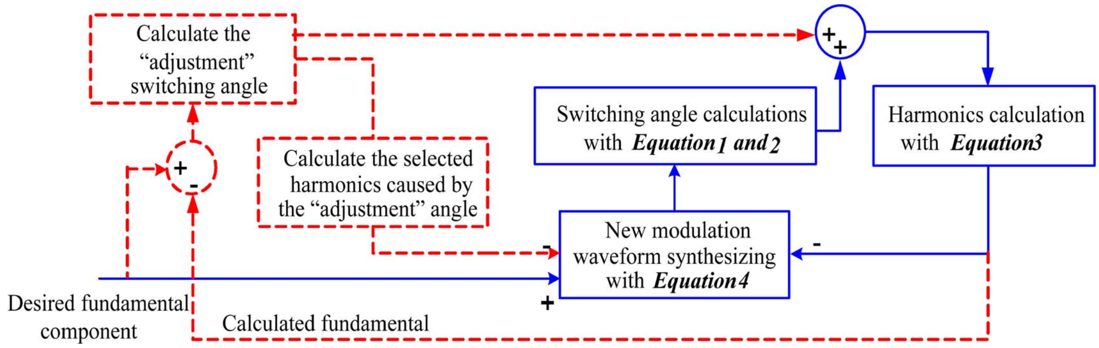

Fig. 6. Modified method with adjustment switching angle for the highest voltage level.

TABLE IV
SAMPLE POINTS FOR SIX (MAXIMUM) LEVEL WAVEFORM WITH THE MODIFIED FOUR-EQUATION METHOD

Reference MI	Resulted MI	Switching Angles (rad.)					Harmonics (%)				
		θ_1	θ_2	θ_3	θ_4	θ_5	5th	7th	11th	13th	17th
0.90	0.90	0.0641	0.1984	0.3395	0.7714	0.5319	0.069	0.0291	0.0375	0.0474	0.0215
0.76	0.76	0.1878	0.3618	0.5922	0.9231	1.091	0	0	0	0	N/A
0.60	0.60	0.1971	0.4689	0.8051	1.1216	N/A	0	0	0	N/A	N/A
0.46	0.46	0.2175	0.5954	1.0522	N/A	N/A	0	0	N/A	N/A	N/A
0.20	0.20	0.3889	1.4961	N/A	N/A	N/A	0	N/A	N/A	N/A	N/A

for staircase generation, there is no additional voltage level for fundamental voltage compensation. Therefore, in this proposed method, the switching angle of the last dc level will be adjusted to achieve the desired fundamental voltage. The "adjustment" of the switching angle can be calculated by

$$\theta_N^* = a\cos\left(\frac{\pi}{4V_{\text{dc}}}(V_F - V_{1N})\right) \tag{13}$$

where V_{1N} is the total fundamental voltage generated by switching angles from θ_1 to θ_m. This "adjustment" angle is used to modify the switching angle for the last voltage level

$$\theta_{N(\text{modified})} = a\cos\left(\cos(\theta_N) + \cos\left(\theta_N^*\right)\right). \tag{14}$$

Therefore, based on the switching angle adjustment for the last dc level, the desired voltage magnitude in the fundamental frequency can be achieved. However, if not compensated, the "adjustment" switching angle would also bring in the additional harmonics in the resulted staircase waveform. Thus, the selected harmonics caused by the "adjustment" angle would also need to be calculated and added to the final modulation waveform. The total process of this modified method is shown in Fig. 6. Based on the final solutions 1) and 2), switching angles, resulted modulation indices, and selected harmonic contents were calculated with the desired modulation index sweeping from 0.2 to 0.9 for the six (maximum) level staircase waveform. For all the tested modulation indices, the resulted modulation index had followed the desired value very well. SHE can be close to 100% except at modulation indices close to 0.9. Some sample points of the calculation results are shown in Table IV. At the first sample point $MI = 0.9$, since no extra voltage is available, the "adjustment" angle in method 2) is used. For the other four points in the table, an additional voltage level shown in method 1) is used.

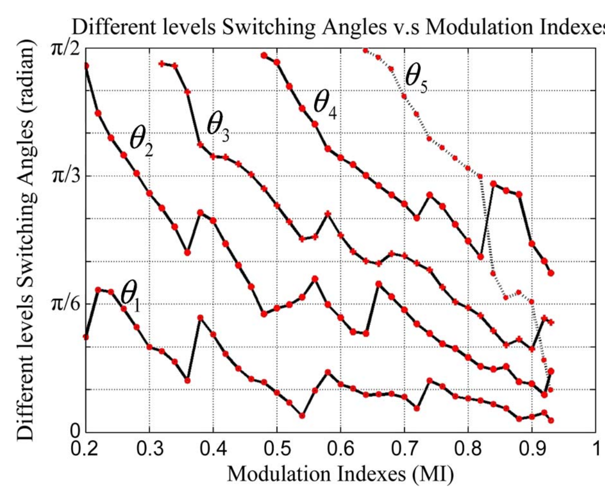

Fig. 7. Optimized switching angles of the proposed method.

The overall optimized switching angles based on the proposed method for the six-level waveform are shown in Fig. 7. The results show that both solutions 1) and 2) work well in terms of achieving the desired fundamental voltage and harmonic elimination. It is noticeable that, with the modification of the switching angle of the highest voltage level, θ_5 sometimes becomes smaller than other switching angles. It may look strange, but this will not cause any problem. In the real inverter, the final voltage is the summation of the voltage from all the dc sources; instead of getting the waveform shown in Fig. 8(a),

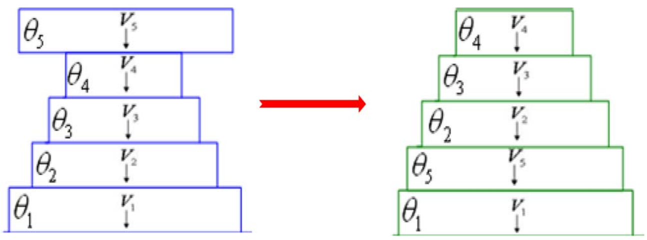

Fig. 8. Explanation of the top switching angle in Fig. 7.

Fig. 9. Cascade multilevel inverters of 1 MVA.

TABLE V
SWITCHING ANGLES CALCULATED FOR $MI = 0.84$

MI=0.84	$\theta_1(rad.)$	$\theta_2(rad.)$	$\theta_3(rad.)$	$\theta_4(rad.)$
	0.05995	0.18863	0.28101	0.36322
	$\theta_5(rad.)$	$\theta_6(rad.)$	$\theta_7(rad.)$	$\theta_8(rad.)$
	0.50503	0.63771	0.87771	1.0889

the output voltage of the inverter would always look like the waveform in Fig. 8(b).

Based on modulation index definition, when the modulation index is less than 0.2, the reference voltage is below the first dc voltage level, and there is no cross point between the dc level and the reference voltage. Therefore, the first switching angle can just be used to generate reference voltage, and no harmonic elimination can be realized. For modulation index larger than 0.9, the reference voltage is much higher than the last dc level. Thus, the modification of the last switching angle is not efficient anymore in achieving good harmonic elimination. All the SHE methods for multilevel inverters are facing similar limitations at very low and very high modulation indices.

V. Experimental Verification

Aside from the simplicity of the algorithm and high precision in harmonic elimination, another advantage of the proposed method is that this method can be easily adopted to calculate the switching angles of staircase waveform with a high number of dc levels. Thus, the verification test was performed with a 17-level cascade inverter shown in Fig. 9. The inverter was designed to deliver 1 MVA at 6000 V. Each phase of the inverter contains four identical cascaded units. Each unit is a three-level diode-clamped H-bridge. Thus, eight switching angles are needed to create maximum 17 voltage levels. To verify the accuracy of the proposed method, tests were done at no load condition. During the tests, dc voltage was fixed at 400 V. Eight switching angles were calculated to generate a 17-level phase voltage. These angles are listed in Table. V. In the calculation results, the aimed harmonics are less than 1 pico p.u. Fig. 10(a) shows the experimental phase voltage. The corresponding fast Fourier transform (FFT) analysis of the waveform is shown in Fig. 10(b). Fig. 11 shows the experimental line–line voltage and its FFT analysis. Both FFT results in Figs. 10 and 11 show that the aimed harmonics are eliminated as expected. In the line–line voltage, tripled harmonics are also canceled. The experimental results verify the proposed method well.

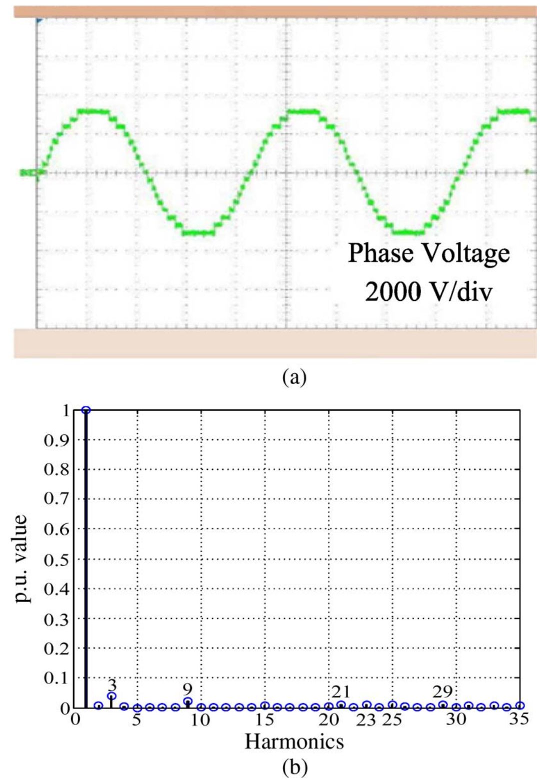

Fig. 10. Output phase voltage at $MI = 0.84$. (a) The phase voltage. (b) FFT analysis results.

VI. Conclusion

In this paper, a simple four-equation-based method has been proposed for SHE in multilevel inverters. The problems of direct implementation of this method are identified and explained. Final solutions are proposed and verified with case study and experimental verification. Comparing with other SHE methods proposed for multilevel inverters, the harmonic elimination method proposed in this paper has the following advantages: 1) Only four simple equations are involved; 2) for different numbers of switching angles, the equations remain the same, and no huge increasing of calculation time is expected when the

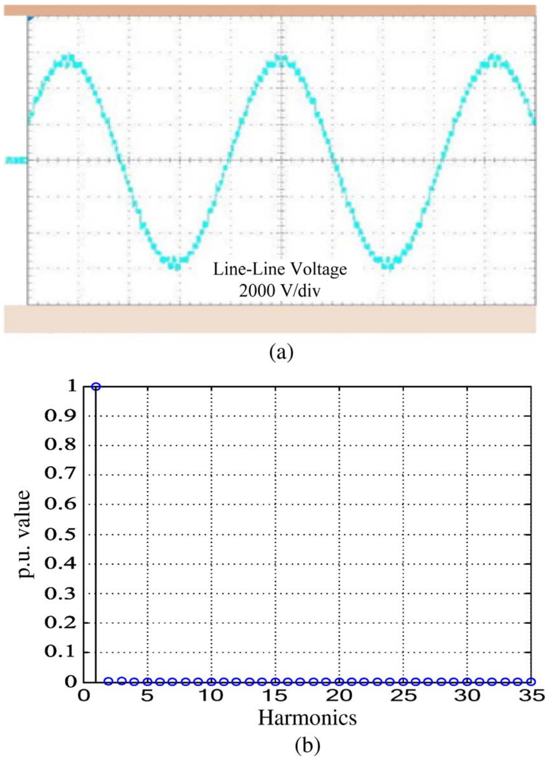

Fig. 11. Output line–line voltage at $MI = 0.84$. (a) The line–line voltage. (b) FFT analysis results.

number of switching angles increases; and 3) in some cases, this method can eliminate more than $N - 1$ harmonics with only a small difference between the desired and resulted modulation indices. This method is not only precise in harmonic elimination but also practical in terms of simplicity and realization by field engineers.

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