Optimal PWM Method based on Harmonics Injection and Equal Area Criteria

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Abstract--- **For high power inverters, Optimal Pulse Width Modulation (OPWM) is the most popular method to reduce the switching frequency and at the same time realize selected harmonics cancellation. Most published methods for optimal PWM are based on solving multiple variable and high order nonlinear equations. This paper proposes a practical harmonics injection and equal area criteria based method to realize optimal PWM. Compared with traditional methods, this proposed method does not involve complex equation groups and is much easier to be utilized in the cases with many switching angles in wide range of modulation indexes.**

Index Terms-- **Equal Area Criteria, Harmonics Injection, Optimal PWM**

I. INTRODUCTION

With the development of different types of distributed generation, such as fuel cells, photovoltaics, and wind turbines, implementations of megawatt level inverters are becoming more popular [1]-[3]. For these high power and possible medium and high voltage level converters, switching loss is as important as power quality. Selective harmonics elimination based on OPWM is the perfect match for these megawatt level inverters in reducing the switching frequency while keeping the THD level under numbers specified by regulations and standards [4].

Selective harmonics elimination was first proposed by Turnbull in 1964 [5]. The basic idea is that the PWM waveform can be expressed in a form of Fourier series. The harmonics contents of this Fourier series can be written as functions of the switching angles in trigonometric terms. Then a multiple variable equation group is formed, with one equation to guarantee the amplitude of the fundamental component, and other equations to eliminate harmonics. A total of N-1 harmonics can be eliminated, where N is the total number of switching transitions [6]-[10].

Earlier methods range from Newton-Raphson methods to linearization for real time calculation [11]-[12]. Some recent works on this topic utilizes advanced algorithms and control theories such as resultant theory, fuzzy logic, and sliding mode control. Online calculations of the switching angels are introduced in [13], [14]. But more or less, all these methods are still based on solving the complex equation groups, which would be quite difficult or even impossible to solve with current computation methods when the number of switching transients is high [15], [16].

To overcome this problem, harmonic injection and equal area criteria based four-equation method has been proposed in [17]. In this solution, regardless how many voltage levels are involved in multilevel inverters, four simple equations are used in the basic method. A full study of this method with equal dc voltage level has been presented recently [18]. This paper shows that the same principle can be used for more harmonics elimination in OPWM for two level inverters.

II. BASIC PROCEDURE OF OPWM IN THE PROPOSED METHOD

In this paper, equal area criteria and harmonics injection based optimal PWM method is proposed. The basic idea is that if a sinusoidal reference waveform is used to generate a series of switching angles based on equal area criteria, the resulted PWM waveform would have both fundamental components and lower order harmonics. Then, if selected lower order harmonics are injected to the original pure sinusoidal reference waveform, the resulted lower order harmonics may cancel out the lower order harmonics generated by the original pure sinusoidal reference. The following is the detailed description of the proposed method.

For the simple case shown in Fig. 1, the harmonics content of the PWM waveform can be calculated from:

$$h_m = \sum_{k=1,2,..,N}^{N} \frac{2V_{dc}}{m\pi}(\cos(m\sigma_{k-1}) - \cos(m\theta_k)) \tag{1}$$

where N is the total number of σ_k and "m" is the order of the harmonics. Based on this single equation, the proposed method would include the following basic procedure shown in Fig. 2:

1) Use a pure sinusoidal waveform and equal area criteria to decide initial angles of θ_k with predefined initial values of σ_k ;
2) Find the lower harmonics content in the resulted PWM waveform with (1);
3) Form a new reference waveform which is defined by

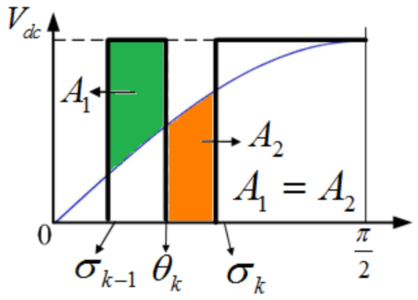


Fig. 1. The illustration of equal area criteria.

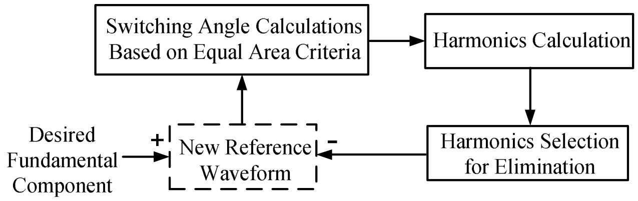


Fig. 2. The diagram showing four-equation method.

$$V_{ref} = V_F \sin(\omega t) - h_{ms} \sin(m\omega t) \tag{2}$$

where h_{ms} is the sum of h_m :

$$h_{ms} = \sum_{i=1,2,\ldots}^{iter} h_{m(i)} \tag{3}$$

4) Use the new reference waveform and equal area criteria to form a new set of σ_k .

5) Repeat step 2) to 4) until the selected harmonics is eliminated. The general equation to calculated the θ_k based on equal area criteria and σ_k is:

$$\begin{aligned}\theta_k = \sigma_k &- \frac{V_F(\cos(\sigma_{k-1}) - \cos(\sigma_k))}{V_{dc}} \\ &- \frac{h_{5s}(\cos(5\sigma_{k-1}) - \cos(5\sigma_k))}{5V_{dc}} \\ &\ldots - \frac{h_{ms}(\cos(m\sigma_{k-1}) - \cos(m\sigma_k))}{mV_{dc}}\end{aligned} \tag{4}$$

From the basic procedure, it is clear that the proposed method is an iteration based method. The initial start point of σ_k is very crucial. Based on the symmetry of the solutions identified in [9], the initial σ_k s can be evenly distributed in the region of 0 to $\pi/2$, specially for medium and high modulation indexes.·

The equation (4) shows that when the equal area criteria is involved, there is defined relationship between σ_k and θ_k . So, the total number of eliminated harmonics is N-1, where N is the total number of σ_k . In the methods based on solving high order nonlinear equations, the total number of eliminated harmonics can be as high as 2N-2. So this is the main drawback of the proposed method. But this can be compensated by using a higher number of N.

Indeed, the advantage of the proposed method is its simplicity in calculations. There is no longer a need to solve high order nonlinear equations, so advance algorithms are no longer needed. In the traditional methods, the number of equations grows with the number of switching angles. It is very difficult to calculate the switching angles when the total number of the switching transients is high. Conversely, the number of equations involved in the proposed method remains the same no matter how many switching angles are involved. Thus, there is no significant increase in calculation time with an increase in the number of switching angles. In general, this method would be more suitable for the case with online calculations of high numbers of switching angles. This paper mainly serves as an introduction of the proposed method. The online calculation would be addressed in the following up papers.

With the basic procedure described above, it is difficult to make sure the resulting fundamental component is same as the desired fundamental component. This is because of possible over modulation caused by the harmonics injection or overlap of switching angles at high modulation index. Thus, fundamental voltage compensation is needed for the proposed method.

The basic approach is to compare the resulting fundamental component and the desired component. Then, based on this difference, a $\Delta\delta$ is calculated to modify the last switching angle that is the nearest to $\pi/2$. This $\Delta\delta$ results in more harmonics. So the resulting additional harmonics are calculated and added to the total harmonics injection through the four-equation method for harmonics cancellation. This "adjustment" angle can be calculated by the following equation:

$$\Delta\delta = a\cos\left(\frac{\pi}{2V_{dc}}(V_F - V_{1N})\right) \tag{5}$$

where V_{1N} is the total fundamental voltage generated by switching angles from θ_1 to θ_N. This "adjustment" angle is used to modify the last switching angle:

$$\theta_{N(\mathrm{mod}\,ified)} = a\cos(\cos(\theta_N) + \cos(\theta_N^*)) \tag{6}$$

Therefore, based on the switching angle adjustment, the desired voltage magnitude in the fundamental frequency can be achieved. The total process of this modified method is illustrated in Fig. 3.

III. CASE STUDY

A case study with 10 θ_k for a single phase case has been performed. Table 1 shows some sample points achieved with

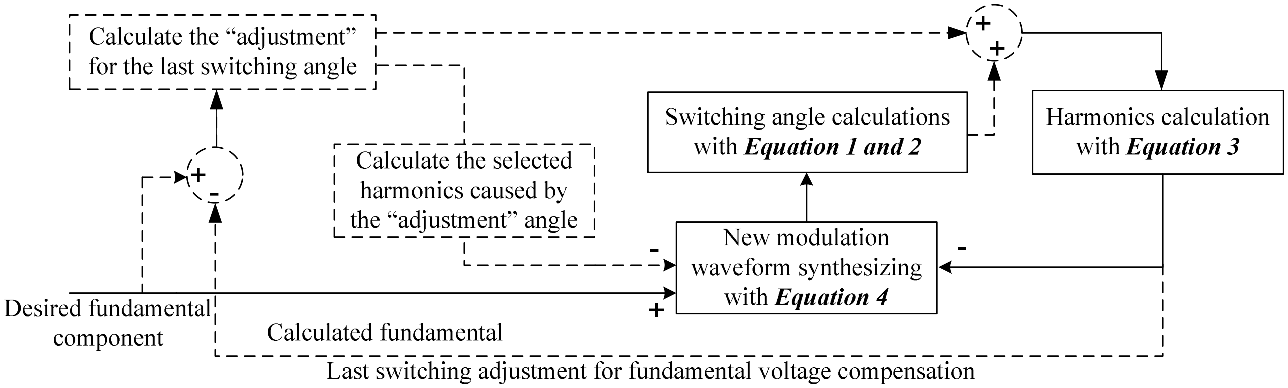


Fig. 3. Modified method with "adjustment" for the switching angle in optimal PWM based on four-equation method.

Table 1. Sample points with proposed method.

Modulation Indices		Delta									
		Delta 1	Delta 2	Delta 3	Delta 4	Delta 5	Delta 6	Delta 7	Delta 8	Delta 9	Delta 10
0.8286		0.1399	0.2696	0.4050	0.5404	0.6825	0.8250	0.9644	1.1049	1.2540	1.4043
0.7402		0.1598	0.3031	0.3808	0.5486	0.6772	0.8676	0.9863	1.1173	1.2295	1.4510
0.6748		0.1273	0.2664	0.4309	0.5762	0.7001	0.8272	0.9796	1.1440	1.2993	1.4274
0.6016		0.1285	0.2742	0.4331	0.5880	0.7055	0.8383	0.9920	1.1573	1.3075	1.4471
0.5246		0.1261	0.276	0.4433	0.6023	0.7118	0.8441	1.0043	1.1763	1.3242	1.4612
0.2672		0.7166	0.8265	0.8995	0.9884	1.0853	1.1685	1.2491	1.3613	1.4206	1.5406
0.1559		0.7437	0.8630	0.9122	1.0250	1.1032	1.1947	1.2726	1.3849	1.4387	1.5595
		Harmonics (%)									
Order of the harmonics		1st	5th	7th	11th	13th	17th	19th	23rd	29th	31st
Modulation Indices	0.8286	100.00	0.00	0.00	0.3066	0.00	0.2383	0.00	0.7344	0.3151	0.0884
	0.7402	100.00	0.00	0.00	0.03	0.00	0.18	0.00	0.23	0.09	0.00
	0.6748	100.00	0.00	0.00	0.08	0.00	0.12	0.00	0.17	0.21	0.01
	0.6016	100.00	0.00	0.00	0.16	0.00	0.13	0.00	0.16	0.23	0.00
	0.5246	100.00	0.00	0.00	0.23	0.00	0.15	0.00	0.26	0.17	0.00
	0.2672	100.00	0.0091	0.0121	0.0158	0.0198	0.0214	0.0259	0.0249	0.0204	0.0114
	0.1559	100.00	0.0007	0.0009	0.0010	0.0013	0.0015	0.0015	0.0013	0.00	0.0010

used to test this method. In this case, δ_k s are simply fixed at points $k \cdot \pi / 20$ and fundamental compensation is used for these sample points. From the harmonics analysis, it can be seen that 9 elected harmonics are eliminated precisely.

IV. SIMULATION RESULTS

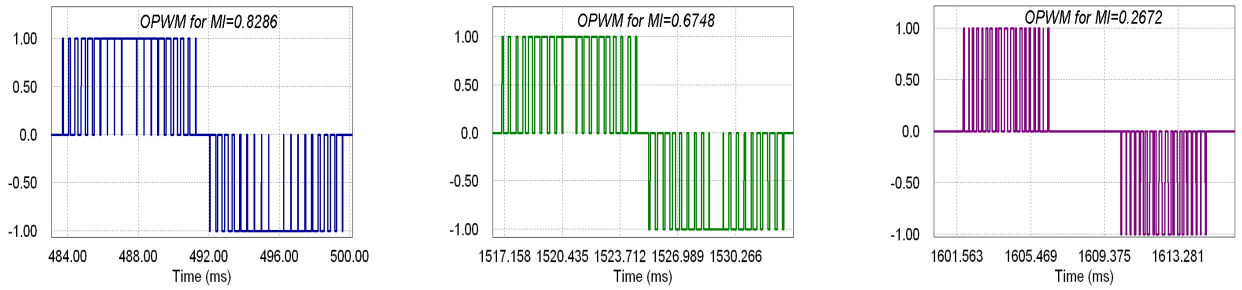
To verify the proposed method for harmonic elimination, the optimal PWM based on four equation method is simulated for three different modulation indices: 0.8286, 0.6748, and 0.2672. The switching angles for these three cases are shown in Table 1. The simulated output voltages for these cases are shown in Fig. 4(a)-(c). Then, the output simulated voltage is decomposed in the selected frequencies for harmonic components analysis. The results in Table 2 shows good selected harmonics elimination for each modulation index.

V. EXPERIMENTAL VERIFICATION

An experimental setup, as shown in Fig. 5, has been utilized to test the sample switching angles from the method. During the test, a DSP controlled 100 kW H-bridge inverter with inductive load was used. The peak current of the load was 30 A.

Fig. 6 shows the voltage and current waveforms achieved from the test at modulation index of 0.6748. Fig. 6(a) is the output voltage at no load condition. Fig. 6(b) is the voltage and current waveform with the inductive load mentioned above. The selected harmonics content of voltage and current shown in Fig. 6(b) is summarized in Table 3.

Comparing Table 3 to the sample points of MI=0.6748 in Table 1 and 2, the experimental results have slight higher harmonics content than the calculation results. This is mainly because of the dead-time and dc voltage fluctuation during the testes.



(a) MI=0.8286

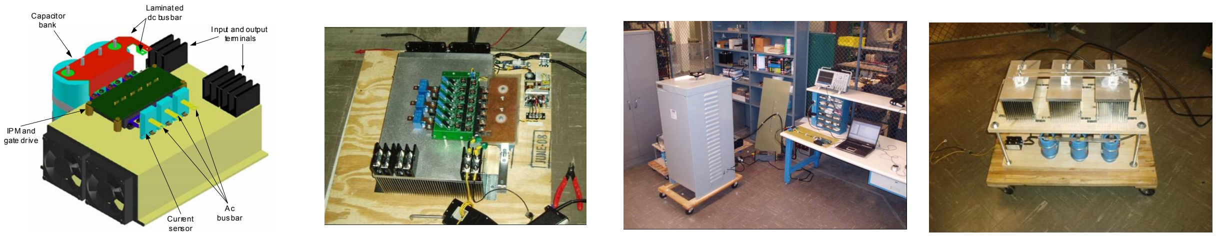
(b) MI=0.6748

(c) MI=0.2672

Fig. 4. Simulated output voltage based on OPWM and four-equation in three different modulation indices.

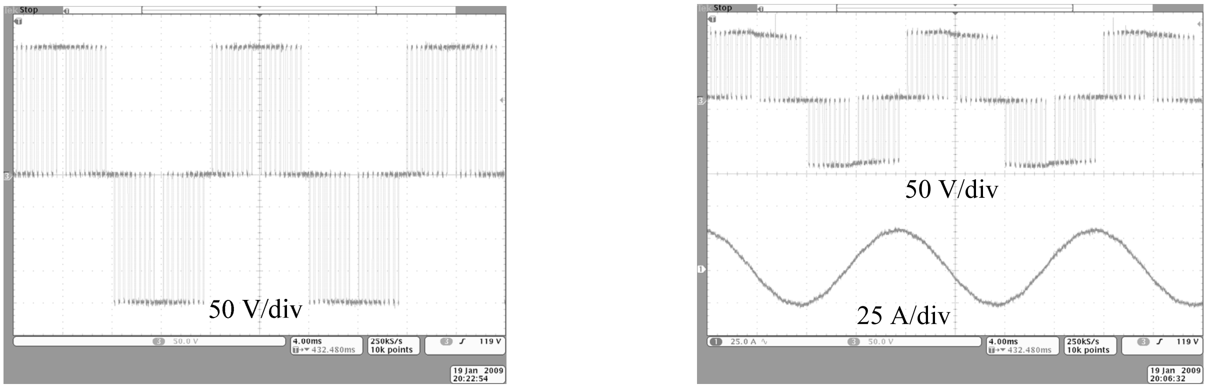
Table 2. Harmonic components for the simulated modulation indices.

		Harmonics (%)									
Order of the harmonics		1st	5th	7th	11th	13th	17th	19th	23rd	29th	31st
Modulation Indices	0.8286	100.00	0.3152	0.6465	0.1324	0.1799	0.3075	0.7742	0.4937	0.5002	0.4827
	0.6748	100.00	0.0915	0.0884	0.2616	0.0430	0.4399	0.0380	0.5783	0.6635	0.0658
	0.2672	100.00	0.1712	0.0871	0.1489	0.0607	0.0563	0.0923	0.2000	0.1049	0.0905



100 kW Inverter Design **Inverter Under Test** **High Power AC Source** **Rectifier (AC to DC)**

Fig. 5. Test setup for optimal PWM in the proposed method.



(a)

(b)

Fig. 6. Experimental results, (a) No load test voltage and, (b) Load test voltage and current waveforms.

Table 3. Harmonics content in the voltage and current for MI=0.6605.

	Harmonics (%)									
Order of the harmonics	1st	5th	7th	11th	13th	17th	19th	23rd	29th	31st
Voltage	100.00	0.5570	0.3759	0.9750	0.3054	0.4229	0.0611	0.0435	0.1410	0.1175
Current	100.00	0.0384	0.0248	0.2703	0.0134	0.0518	0.0187	0.0360	0.0437	0.0394

VI. CONCLUSIONS

In this paper, the equal area criteria and harmonics injection method for optimal PWM is introduced. Then, through four simple equations, switching angles are calculated for harmonic cancellation. The proposed method is simulated for three different modulation indices and verified by experimental tests. In the real test, there is a small difference between desired and resulted harmonics compensation, which is mainly caused by deadtim and dc voltage fluctuation. Compared with earlier published methods, which are often based on solving multiple variable nonlinear equation groups, the proposed method is much simpler in algorithm, and more suitable for the cases with high numbers of switching angles.

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