Selective Harmonic Elimination for Multilevel Inverters with Unbalanced DC Inputs

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***Abstract*- Selective harmonics elimination for the staircase voltage waveform generated by multilevel inverters has been widely studied in the last decade for medium and high voltage applications. Most published methods on this topic are used for balanced multilevel inverters with the same dc voltage magnitude. In the proposed method, equal area criteria and harmonics injection are used for totally unbalanced multilevel inverters. Regardless of how many voltage levels are involved, only four modified equations are proposed in the basic method. To eliminate the harmonic components in unbalanced conditions, the problems of the basic method are discussed for a wide range of modulation indices. Then, a full set of solutions is proposed based on the combination of optimal PWM and the four-equation based method for a wide range of modulation indices in unbalanced conditions.**

***Keywords:* Equal Area Criteria, Modulation Index, Pulse Width Modulation, Total Harmonic Distortion, Unbalanced Multilevel Inverters.**

I. INTRODUCTION

The recent developments of Hybrid Electric Vehicles (HEVs) has provided great opportunities for the implementation of medium and high power inverter/converter circuits and associated control strategies [1]. In large electric drives with high power demands, such as heavy duty military trucks, multilevel inverters are the natural choices because of higher efficiency, lower dv/dt, and the utilization of lower voltage rated devices [2],[3].

A typical multilevel inverter utilizes voltage levels from multiple dc sources of the same magnitude. These dc sources can be isolated as in cascade multilevel structures or interconnected as in diode clamped structures. In most published multilevel inverter circuit topologies, the dc sources in the circuits need to be maintained to supply identical voltage levels. Based on these identical voltage levels, with proper control of the switching angles, the switches only need to switch on and off once during a fundamental cycle to achieve the desired staircase waveform; thus, the switching loss of the device is reduced to minimum. However, with reduced switching frequencies, even with additional voltage levels, low frequency harmonics are generated in the staircase voltage [4], [5].

Based on Fourier expansion of the staircase waveform generated by multi-level inverters, several methods have been proposed to eliminate selected harmonic components [6], [7]. In most of these methods, for a higher level of multilevel inverters, the order of the equation and the number of the variables both increase with the voltage levels. Therefore, for a high number of switching angles, finding solutions for these equations would become difficult and often involve advanced mathematical algorithms on existing computer algebra software tools [8].

The harmonic injection and equal area criteria based four-equation method was proposed in 2005 [9]. Regardless of how many voltage levels are involved in multilevel inverters, four simple equations are used in the basic method. A full study of this method with an equal dc voltage level has been presented recently [10]. This paper shows that the same method can be adopted to be used for harmonics elimination in multilevel inverter converters with unbalanced dc inputs.

In this paper, first, the basic four-equation method is adopted for multilevel inverters with unbalanced dc voltage levels. Then, the problem of the proposed method and proposed solutions for low and high modulation indices are discussed. Finally, for low modulation indices where harmonic elimination is limited due to the small number of voltage levels, a new method combining a modified version of the four-equation based method and the basic idea of optimal PWM is proposed. The full analysis of the combined method and extended results will be shown in a follow up paper.

II. THE PROPOSED METHOD FOR UNBALANCED DC INPUTS

The basic idea of the four-equation method for harmonics elimination in a multilevel inverter with unbalanced dc levels is shown in Fig. 1. The harmonics elimination is realized with harmonics injection to the modulation waveform and switching angle calculation based on equal area criteria. The associated algorithm is summarized by the following:

Equation 1: Based on Newton-Raphson method, this equation is used to find numerical solutions for the junction points of reference waveform and voltage level:

$$\delta_k = \arctan\left(\frac{\sum_{i=1}^{k} V_{dc(i)} + h_5 \sin(5\delta_k) \ldots h_m \sin(m\delta_k)}{V_F \cos(\delta_k)}\right) \qquad (1)$$

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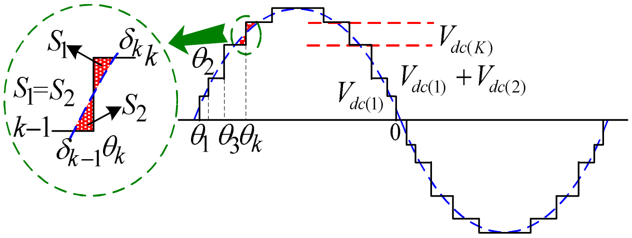


Fig. 1: Equal area criteria in multilevel inverters with unbalanced dc levels.

Equation 2: Based on the junction points, switching angles are calculated by this equation:

$$\theta_k = 1/V_{dc(k)} * (\sum_{i=1}^{k} V_{dc(i)}\delta_k - \sum_{i=1}^{k-1} V_{dc(i)}\delta_{k-1} + V_F(\cos(\delta_k) - \cos(\delta_{k-1})) - \frac{h_5}{5}(\cos(5\delta_k) - \cos(5\delta_{k-1}))\ldots - \frac{h_m}{m}(\cos(m\delta_k) - \cos(m\delta_{k-1}))) \quad (2)$$

Equation 3: The equation is used to define harmonic components by switching angles for different frequencies:

$$h_m = \sum_{k=1,2,..,N}^{N} \frac{2V_{dc(k)}}{m\pi}(\cos(m\theta_k) - \cos(m(\pi - \theta_k))) \quad (3)$$

Equation 4: This equation is used to generate a new reference waveform:

$$V_{ref} = V_F \sin(\omega t) - h_{ms}\sin(m\omega t) \quad (4)$$

Where, h_{ms} is the sum of h_m calculated in every iteration:

$$h_{ms} = \sum_{i=1,2,\ldots}^{iter} h_{m(i)} \quad (5)$$

To identify possible problems of this method for harmonic elimination, the group of equations is applied to the 6-level waveform with different dc magnitudes as shown in Table.1, for modulation index changing from 0.16 to 0.94. The modulation index is found?? by:

$$MI = \frac{V_F}{\frac{4}{\pi}\sum_{i=1}^{N} V_{dc(i)}} \quad (6)$$

where, V_F is the reference ac voltage in the output, N is the number of dc levels, and $V_{dc(i)}$ is the dc magnitude for each voltage level in multilevel inverter output waveform.

The main problem identified in this process is the amplitude difference between the desired and resulted fundamental voltages. With the direct implementation of the proposed method, the fundamental voltage of the staircase waveform often diverts from the desired value, as shown in Table 2.

Table. 1: Five different dc levels for staircase waveform.

DC Levels	V_1	V_2	V_3	V_4	V_5
P.U	1	1.15	0.95	1.05	0.85

III. THE MODIFIED METHODS FOR DIFFERENT MI

To solve the above mentioned problem, modulation indexes are categorized in two groups for low modulation indices at which an extra dc level is available and high modulation indices, where no extra voltage level is available, as shown in Fig. 2.

A. Low Modulation Indices with Extra Voltage Levels

In these conditions, fewer dc levels are utilized to synthesize the staircase waveform. Therefore, an extra voltage is available and can be used for fundamental voltage compensation. Based on this idea, an extra switching angle in the extra voltage level is calculated to achieve desired fundamental voltage. However, this switching angle generates additional harmonic components. So these additional harmonics are also added to the reference waveform that is used to calculate the other switching angles. Then, the generated harmonics by the extra switching angle is compensated by other switching angles. In these conditions, two equations are used to calculate the additional switching angle:

1) First, the fundamental voltage based on switching angles from θ_1 to θ_m is calculated with the following equation:

$$V_{1m} = \sum_{i=1}^{m} \frac{4V_{dc(i)}}{\pi}\cos(\theta_i), \qquad m < N \quad (7)$$

2) Then, additional switching angle is calculated to compensate the difference between V_{1m} and fundamental voltage:

$$\theta_{m+1} = a\cos(\frac{\pi}{4V_{dc(m+1)}}(V_F - V_{1m})) \quad (8)$$

This idea for low modulation indices is shown in Fig. 3. Thus, based on four-equation based method, and through (7) and (8), harmonic components are eliminated by "m" switching angles and the reference fundamental voltage is compensated adequately by θ_{m+1}. The additional process used for voltage compensation in "m+1" level is shown in dotted line.

B. High Modulation Indices with No Extra Voltage Levels

At this condition, all dc levels are used for staircase generation and there is no additional voltage level for fundamental voltage compensation. Therefore, in the proposed method, again, two equations are used for fundamental voltage compensation:

Table 2. Sample points from the direct implementation of the basic four-equation method.

Reference MI	Resulted MI	Switching Angles (rad.)					Harmonics (%)					
		θ_1	θ_2	θ_3	θ_4	θ_5	1st	5th	7th	11th	13th	17th
0.92	0.7878	0.1238	0.3448	0.5465	0.8532	1.1308	85.6259	0.0035	0.0004	0.0041	0.0119	0.0176
0.87	0.7877	0.1240	0.3450	0.5468	0.8535	1.1308	90.5357	0.0035	0.0008	0.0043	0.0085	0.0086
0.78	0.68	0.1503	0.3419	0.6173	1.0160	N/A	87.1772	0.0004	0.0003	0.0005	0.0030	N/A
0.65	0.5075	0.1749	0.4731	0.9804	N/A	N/A	78.0698	0.0010	0.0030	0.0066	N/A	N/A
0.52	0.5074	0.1750	0.4732	0.9806	N/A	N/A	97.5810	0	0.0006	0.0007	N/A	N/A

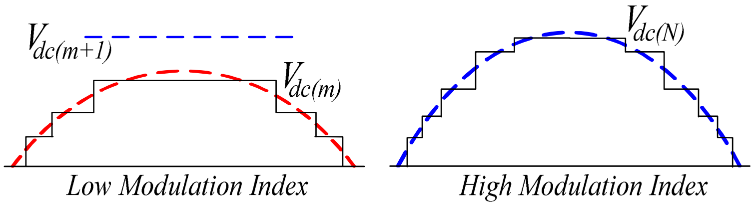


Fig. 2: Two groups of modulation indices in multilevel inverters with unbalanced dc levels.

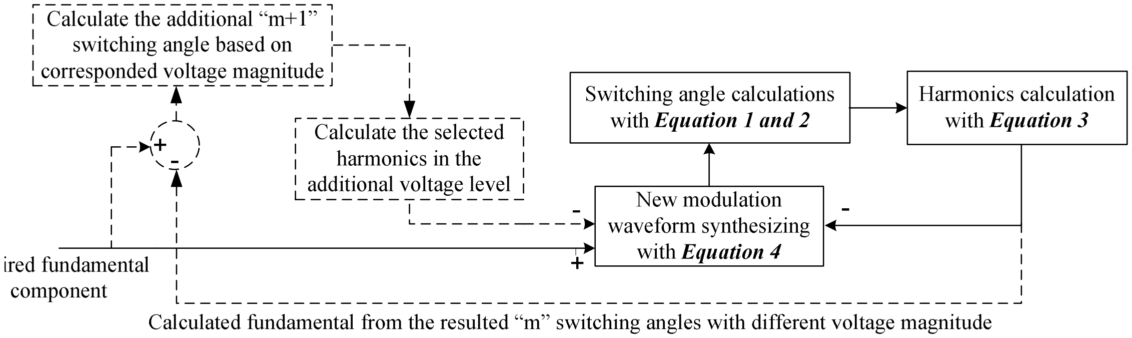


Fig. 3. Modified method with "additional" switching angle in unbalanced multilevel inverters.

1) An "adjustment" switching angle is calculated to achieve fundamental voltage by the following equation:

$$\theta_N^* = a\cos\left(\frac{\pi}{4V_{dc(N)}}(V_F - V_{1N})\right) \qquad (9)$$

where V_{1N} is the total fundamental voltage generated by switching angles from θ_1 to θ_N.

2) This "adjustment" angle is used to modify the switching angle for the last voltage level:

$$\theta_{N(\text{modified})} = a\cos(\cos(\theta_N) + \cos(\theta_N^*)) \qquad (10)$$

Therefore, based on the switching angle adjustment for the last dc level, the desired voltage magnitude in the fundamental frequency can be achieved. However, the "adjustment" switching angle also causes new harmonic components used in the final modulation waveform in equal area criteria. The total process for this adjustment is shown in Fig. 4. Thus, the last switching angle, θ_N, is used, both for harmonic elimination and fundamental voltage compensation. The additional process for this adjustment in four-equation based method is shown in dotted line.

In Table 3, some sample points are shown for low and high modulation indices in the proposed method. As mentioned in the results, harmonic components are eliminated successfully and fundamental voltage is generated precisely.

IV. OPTIMAL PWM AND FOUR-EQUATION METHOD COMBINED SOLUTION

At low modulation indices, fewer number of dc levels is available for staircase waveform generation, thus there is decreasing of switching angle numbers. Therefore, with four-equation based method and Fourier analysis, staircase waveform would have more harmonic components. To overcome this problem, in each dc level, the number of switching angles can be increased to eliminate more harmonic components. In [13], it has been approved that the four-equation method can also be adopted to calculate the switching angles of single level optimal PWM. Thus, the ongoing research is to use the four-equation based optimal PWM method for waveform with three levels. In this method, based on available dc voltage levels in multilevel inverters, the number of switching angles for each level can be increased. Thus, more harmonic components can be eliminated specially in low modulation indices. This approach for multilevel inverters with unbalanced dc inputs is shown in Fig. 5.

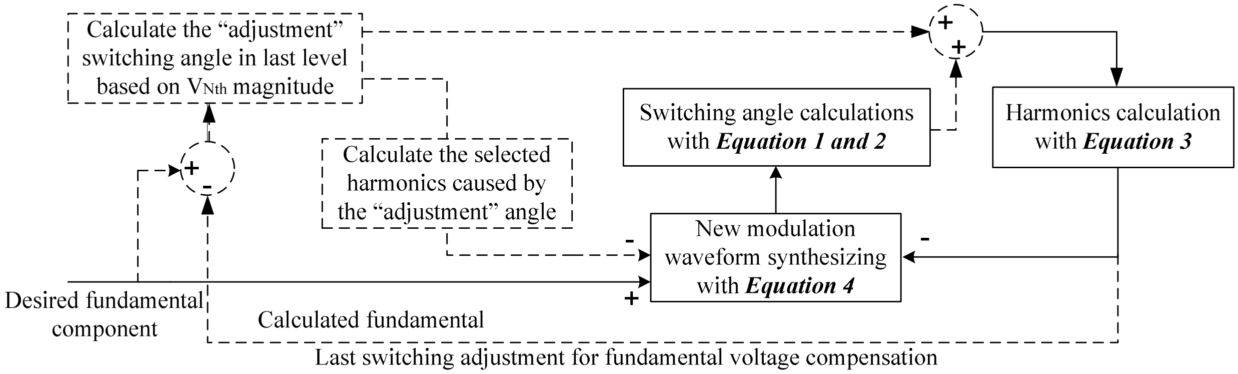


Fig. 4. Modified method with "adjustment" switching angle for the highest voltage level in unbalanced multilevel inverters.

Table 3. Sample points based on the modified four-equation method for different modulation indices.

Reference MI	Resulted MI	Switching Angles (rad.)					Harmonics (%)					
		θ_1	θ_2	θ_3	θ_4	θ_5	1st	5th	7th	11th	13th	17th
0.90	0.90	0.0729	0.1782	0.3897	0.7798	0.5011	100	0.6232	0.9115	0.4412	0.2829	0.4752
0.78	0.78	0.1365	0.3538	0.5692	0.8767	1.1353	100	0	0	0	0	N/A
0.65	0.65	0.1683	0.4020	0.7310	1.0760	N/A	100	0	0	0	N/A	N/A
0.48	0.48	0.1879	0.5898	1.0630	N/A	N/A	100	0	0	N/A	N/A	N/A
0.21	0.21	0.4886	1.4250	N/A	N/A	N/A	100	0	N/A	N/A	N/A	N/A

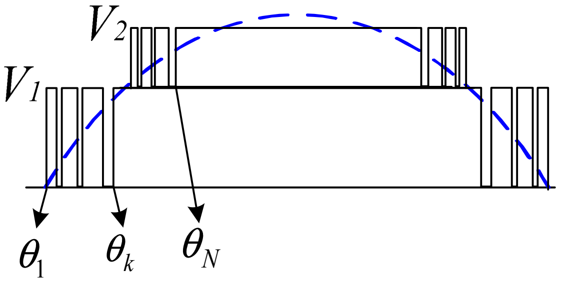


Fig. 5. Optimal PWM in low modulation indices in unbalanced multilevel inverters.

In the proposed method, the number of switching angles used for each level is not limited in theory. Thus, the harmonic elimination can be proposed very efficiently, even for low modulation indices. In this condition, the advantages of optimal PWM for selected harmonic elimination can be applied for unbalanced multilevel inverters. However, based on multilevel inverters applications for medium and high voltages, the switching frequency and number of switching angles applied for inverters is limited based on power electronic switches and power losses.

In Table 4, the proposed method is applied for two cases in low modulation indices when three and five dc levels are available. In the first case with three dc levels, for each positive or negative polarity, one level is available. Thus, five switching angles are used in same level. In the second case, five levels are used for multilevel inverters. Therefore, five switching angles are applied in two dc levels for harmonic elimination. In both cases, five harmonic components can be eliminated that verifies the advantage of the proposed method in low modulation indices for multilevel with unbalanced dc voltages.

V. SIMULATION RESULTS

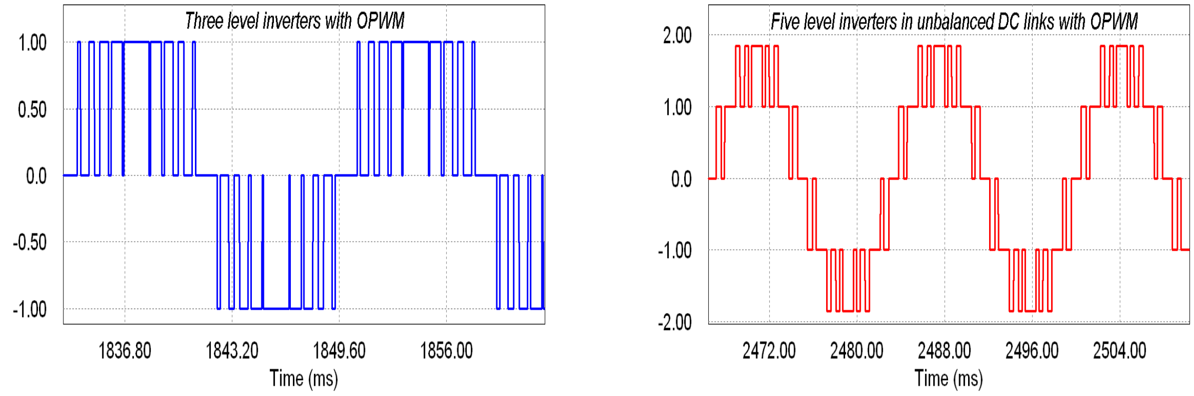
To verify this improvement for harmonic elimination, the multilevel inverter with unbalanced dc levels is simulated in low modulation indices. Then the switching angles, shown in Table 4, are applied to switches in the inverters. Three and five voltage levels for unbalanced input dc are shown in Fig. 6(a) and (b). The output simulated voltage is decomposed in the selected frequencies for harmonic components. These components are shown for two modulation indices in Table 5. As mentioned in Table 5, the selected harmonic components are minimized successfully in the simulated multilevel inverters that verify advantages of using OPWM in four-equation method for low modulation indices.

VI. CONCLUSION

In this paper, a simple method using equal area criteria and harmonic injection is used to eliminate selected harmonic components for multilevel inverter with unbalanced dc inputs. To precisely generate output voltage in the fundamental frequency, for low and high modulation indices, additional angle and adjustment angle are used respectively. However, in four-equation based method, for each dc level just one switching angle is used. Thus, for low modulation indices that few numbers of switching angles are available, the number of eliminated harmonic component is limited. In the proposed method, optimal PWM and four-equation based method are combined together to overcome this limitation in multilevel inverters in low modulation. Then, the number of switching angles and corresponded number of eliminated harmonics can be increased significantly.

Table 4. Sample points based on the OPWM and four-equation based method for low modulation indices

# levels	Modulation Index	Switching Angles (rad.)					Harmonics (% based on fundamental output)					
		θ_1	θ_2	θ_3	θ_4	θ_5	1st	5th	7th	11th	13th	17th
3	0.1335	0.2436	0.5248	0.7490	1.0203	1.3081	100.00	0.00	0.9372	0.00	1.7404	1.2445
	0.1429	0.2453	0.5034	0.7454	1.0070	1.2847	100.00	0.00	0.4284	0.7955	0.00	1.3125
5	0.1885	0.3547	0.5497	1.0294	1.1831	1.3402	100.00	0.00	0.00	0.00	0.00	0.00
	0.2215	0.3520	0.5486	1.0309	1.1774	1.3426	100.00	0.00	0.00	0.00	0.00	0.00
	0.2513	0.1721	0.4602	0.8399	1.1484	1.3873	100.00	0.00	0.00	0.00	0.00	0.00



(a) Three level inverters

(b) Five level inverters with unbalanced DC links

Fig. 6. Simulated output voltage based on OPWM and four-equation based method in multilevel inverters with unbalanced DC links.

Table 5. Simulation results for output voltage and selected harmonic components on different modulation indices.

Modulation Index	Switching Angles (rad.)					Harmonics (% based on fundamental output)					
	θ_1	θ_2	θ_3	θ_4	θ_5	1st	5th	7th	11th	13th	17th
0.1429	0.2453	0.5034	0.7454	1.0070	1.2847	100.00	0.0862	0.4693	0.1041	0.7968	1.1643
0.2513	0.1721	0.4602	0.8399	1.1484	1.3873	100.00	1.1591	0.04131	0.4610	0.3469	0.0605

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