

# The Analysis of DC-DC Converter Topologies Based on Stackable Voltage Elements

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**Abstract** – In this paper, a method for generating non-isolated dc/dc converter topologies is summarized. This method utilizes the stackable voltage stiff elements in a dc/dc converter and arranges them to create new usable voltage levels. By defining the input and output for different voltage stiff elements, a comprehensive set of converters can be created. The canonical first order switching cell, along with a proposed third order switching cell, are used to help analyze the first and third order converter topologies. Detailed analysis is provided using the two switching cells. Finally, transformer-less multiple-inputs converters are also analyzed using both the two switching cells and the stackable voltage element concept.

**Index Terms**—Canonical switching cell, third order switching cell, stackable voltage elements.

## I. INTRODUCTION

Buck, boost, buck/boost, SEPIC, Cuk, Luo, Z-Source...over the years, many types of dc/dc converter topologies have been proposed. One general trend of the circuits is increased component count and relatively more complicated control strategies. Increased component count usually means higher cost and lower reliability. With the wide implementation of renewable energy sources and rapid progress in electrification of personal transportations, there is an ongoing demand for simple, cost efficient and reliable dc/dc converter topologies. Now is a good time to examine and re-think existing dc/dc converters to identify new circuit topologies which do not add extra components.

In 1991, Enslin proposed a novel converter for photovoltaic panels aimed at increasing the efficiency [1]. The idea of this converter is to stack the input capacitor with the output capacitor. In this way, only part of the load current will flow through power electronics circuits and the total efficiency was expected to be higher. In January 2009, two papers on the same circuit concept were published by Dehbonei [2], [3]. The detailed analysis of three basic dc/dc converters using the “stacked” concept was presented. However, by stacking the input and output voltage together, a new converter topology is created and hence it is not accurate to compare the efficiency of the derived circuit with the traditional circuit upon which it is based.

From a historical point of view, Tymerski [4] treated converters as three-terminal devices with one pin from both the input and output connected together. Using this method, one basic converter can generate a total of six converter

topologies. The authors then applied this concept to different topologies and derived a group of converters, in which the one mentioned before is included.

The method mentioned in [4] is effective in deriving new topologies, but the author’s treatment of the original converter as a three terminal device is not inclusive. In fact, many converters may not have this trait where the input and output share a common terminal.

The purpose of this paper is to analyze the problem based on the operation principle of switched mode converters and to present a systemic way of finding stackable voltage levels contained in the circuit. Thus enabling the circuit designer to create alternative topologies based on the combination of those stackable elements.

## II. GENERAL METHOD

Numerous converter topologies can be derived by rearranging components positions and interconnections, but only a small number of tangible converters can be realized with this approach. To discover all the usable converter topologies without analyzing numerous trivial permutations, one should restrict the topologies according to the basic operating principles of switched mode power converters.

For voltage source converters, the components that define the voltage transfer ratio are the inductors. With a stable steady state inductor current at different switching states, one voltage equation can be written for each inductor and the voltage transfer ratio of the converter can be determined by these voltage equations.

Starting from the inductors, the rules for workable switched mode dc/dc converter can be summarized as following:

1. One terminal of the inductor has to be connected to at least two switches directly or indirectly through voltage stiff elements.
2. Under no circumstances should two inductors be connected in series.
3. Voltage stiff element must not be placed in parallel with a non-stiff voltage element.

Voltage stiff elements include capacitors, voltage sources and the output voltage (load). Non-stiff voltage elements include switches and inductors or any combination of one of them with voltage stiff elements.

Rule 1 reflects the fact that in switched mode converters, each inductor should have at least two branches allowing

current to flow in two or more routes at different switching states. Rule 3 is used to find all possible voltage stiff elements in the circuit. These voltage stiff elements can be used individually as inputs or outputs or stacked together to generate additional voltage levels, which can then be utilized.

The following sections will analyze the simple first order and third order dc/dc converters. Classification of converters follows the method presented in [4]. A first order converter is the one where only one inductor presents in the circuit and no capacitors exist, aside from the input/output capacitors. A third order converter contains two inductors and a single capacitor, ignoring the input/output capacitors.

### III. FIRST ORDER CASE

The well known canonical switching cell shown in Fig.1 can be used for analyzing first order converters.

There are two switches connected to the inductor and the inductor current has two different routes at the two switching states. From the inductor point of view, changing the switching state connects two different voltages to the inductor. During the steady state operation, the integration of inductor voltage should be equal to zero.

Three components and four connection points are included in this circuit structure, using rule 3, it is easy to calculate the possible places to put in voltage stiff elements. For four points, there are a total of  $C_4^2 = 6$  possible connections and  $C_4^2 - 3 = 3$  possible voltage stiff elements (6 all possible connections minus 3 positions with one non-stiff voltage components). The non-stiff voltage elements are named as  $V_1$ ,  $V_2$  and  $V_3$  and shown in Fig.1 (b).

If the on time for  $S_1$  is defined as  $d$ , then for the inductor:

$$V_1 \times d + V_2 \times (1-d) = 0 \Rightarrow \frac{V_2}{V_1} = -\frac{d}{1-d} \quad (1)$$

$$V_3 = V_1 - V_2 = \frac{1}{1-d} V_1 \quad (2)$$

It is easy to see the first order converter has three voltage stiff elements connected head to tail with predefined relationships (1) and (2). The sum of any two of them is equal to the negative of the other one. So although the voltage stiff elements can be stacked together, no extra voltage levels are achieved besides  $V_1$ ,  $\frac{1}{1-d} V_1$  and  $\frac{d}{1-d} V_1$ , if polarity is omitted.

By assigning the input and output to the voltage stiff elements, six converter topologies could be achieved when polarity is not considered. The voltage transfer ratio of these converters are:  $\frac{1}{1-d}$ ,  $1-d$ ,  $d$ ,  $\frac{1}{d}$ ,  $\frac{d}{1-d}$  and  $\frac{1-d}{d}$ . Since

the difference between  $1-d$  and  $d$  is just the way the switches are defined, thus only three distinct voltage transfer ratios exist. And they are the classic three converters: buck converter, boost converter and buck-boost converter. If one considers the polarity, three new converters can be achieved by making the input and output polarity both reversed.

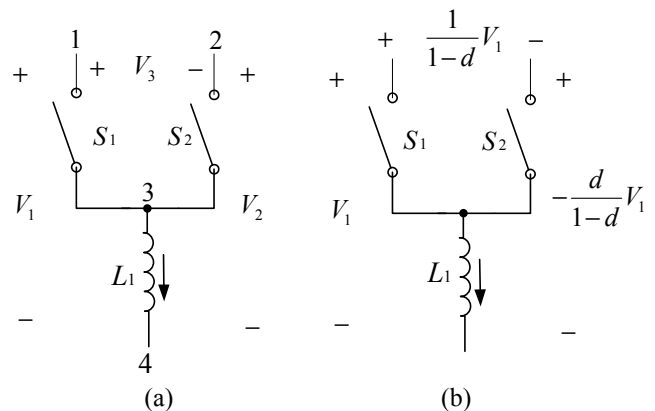


Fig. 1 (a) The canonical switching cell (b) The voltage relationship of different voltage stiff elements.

The voltage and current stress of the switches can also be calculated:

$$V_{S1} = V_{S2} = V_3 = \frac{1}{1-d} V_1 \quad (3)$$

$$I_{S1} = I_{S2} = I_{L1} \quad (4)$$

So for all the topologies that can be derived, the voltage and current stress remain the same.

The six derived first order dc/dc converter topologies have essentially the same transfer functions as those of the classic buck, boost and buck-boost converters and the stress of switches is the same as the classic ones. However, the extra topologies do provide alternative choices to the classic three.

One example is the new boost converter shown in Fig.2. Traditional boost convert use  $-V_1$  as input and  $-V_3$  as output while this one use  $V_1$  as input and as  $V_3$  output. The particular advantage for this converter is that the output capacitor has a smaller voltage stress  $dV_1/(1-d)$  compared to  $V_1/(1-d)$  for the classic circuit. The down side is that the input current is not continuous.

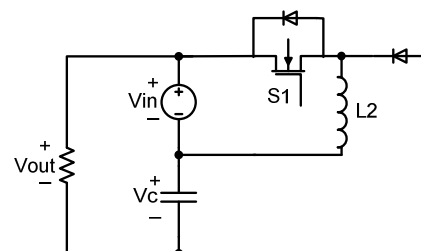


Fig. 2 The new boost converter.

#### IV. THIRD ORDER CASE

From the first order case it can be seen that one inductor needs two switches to provide two alternative conduction paths for different switching states. For the third order case, there are two possible structures. First, one can simply use two first order canonical switching cells in series, in which case the inductor-to-switch ratio is 1:2. A second approach is to use an extra energy storage device (capacitor) as a common intermediate stage and have the two inductors share two switches through this capacitor. This yields two cases for third order converters: one with 4 switches and another with 2 switches. The first case is trivial and will not be discussed here. So only the converters with two inductors, two switches and one capacitor will be discussed here.

##### A. The third order switching cell.

Applying rule 1 and 2 to the case with two inductors, two switches and one capacitor, the only result enables each inductor to have two routes is to form a structure shown in Fig. 3, which is consistent with the given definition for a third order switching cell.

A third order switching cell contains six nodes. Using rule 3 mentioned before, there are totally  $C_6^2 - 4 - 4 = 7$  possible positions for voltage stiff elements (fifteen total possible positions minus four voltage non stiff elements and minus those four combined with the intermediate capacitor voltage  $V_5$ ).

Four voltage stiff elements could be put around the structure:  $V_1$ ,  $V_2$ ,  $V_3$  and  $V_4$ . The middle capacitor has voltage  $V_5$ . Stacking together  $V_1$  with  $V_2$ , and  $V_2$  with  $V_3$  provides two more achievable levels.

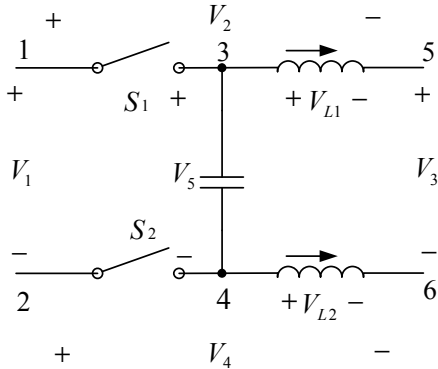


Fig. 3. The third order switching cell.

For the two switching state:

1. When  $S_1$  is closed, and  $S_2$  is open:

$$V_{L1} = V_2 \quad (5)$$

$$V_{L2} = V_2 + V_3 - V_5 \quad (6)$$

2. When  $S_1$  is open,  $S_2$  is closed

$$V_{L1} = V_4 + V_5 - V_3 \quad (7)$$

$$V_{L2} = V_4 \quad (8)$$

When the two inductors are at steady state, the voltage integration should be zero:

$$V_2 \times d + (V_4 + V_5 - V_3) \times (1-d) = 0 \quad (9)$$

$$(V_2 + V_3 - V_5) \times d + V_4 \times (1-d) = 0 \quad (10)$$

So the describing equations for third order switching cells can be written as:

$$\frac{V_4}{V_2} = -\frac{d}{1-d} \quad (11)$$

$$V_3 = V_5 \quad (12)$$

$$V_1 = V_2 + V_3 - V_4 = \frac{1}{1-d} V_2 + V_3 \quad (13)$$

From (11) (12) and (13), it can be seen that there are three groups of voltage stiff elements and two of them are independent:  $V_2$  and  $V_4$  is one group containing duty ratio information,  $V_3$  and  $V_5$  is another group with the same voltage,  $V_1$  contains information from two other groups.

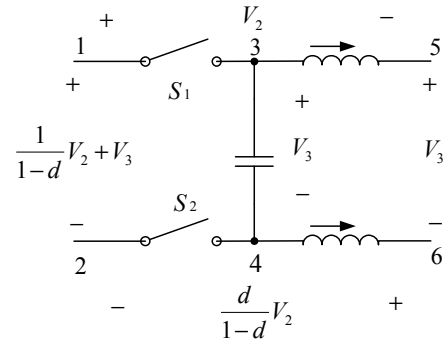


Fig. 4. The relationship of different voltage stiff elements in the third order switching cell.

Since there are two groups of voltage stiff elements, there are also two operation modes: one input mode and two inputs mode.

##### B. One input condition.

In this case, because the  $V_3$  and  $V_5$  do not contain duty ratio information, they cannot be used directly as input or output and they have to be given a definite value by adding extra connections in the circuit. The added connection is a short circuit of one of the 7 available voltage stiff elements. This provides a relationship between the two independent groups.

$V_2$  and  $V_4$  have the duty ratio information so they cannot be shorted. By shorting  $V_3$  (between point 5 and 6), the circuit will be reduced from a third order switching cell to a first order circuit. So the only possible locations for short

circuits are between 1 and 2 ( $V_1$ ), 1 and 6 ( $V_2+V_3$ ), or 2 and 5 ( $V_3+V_4$ ). So there are a total of three possible conditions.

1. Points 1 and 2 are short circuited.

When points 1 and 2 are connected together,  $V_1$  equals 0.

$$V_1 = \frac{1}{1-d}V_2 + V_3 = 0 \Rightarrow V_3 = -\frac{1}{1-d}V_2 \quad (14)$$

This is indeed a three-terminal circuit with three available voltage levels  $V_2$ ,  $\frac{1}{1-d}V_2$  and  $\frac{d}{1-d}V_2$ .  $V_3$  is the high side voltage stiff element,  $V_2$  and  $V_4$  are the two low voltage stiff elements. Based on this relationship, six different converter structures can be built, with the voltage transfer ratio same as the classic buck, boost and buck-boost converter. One example is the Cuk converter in which  $-V_2$  is the input and  $-V_4$  is the output and continuous input and output current can be achieved.

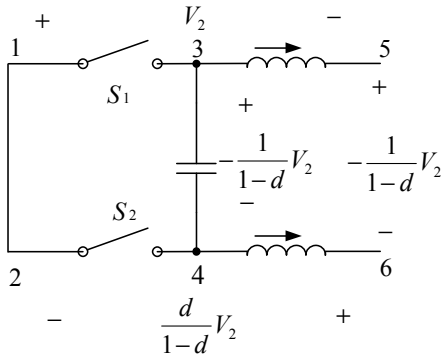


Fig. 5. The third order switching cell with  $V_1$  short circuited.

2. Points 1 and 6 are short circuited.

If points 1 and 6 are connected together, the sum of  $V_2$  and  $V_3$  is equal to 0.

$$V_2 + V_3 = 0 \Rightarrow V_3 = -V_2 \quad (15)$$

$$V_1 = \frac{1}{1-d}V_2 + V_3 = \frac{d}{1-d}V_2 \quad (16)$$

This, however, is not a three terminal circuit, because four voltage stiff elements are connected head to tail. The achievable voltage levels, omitting the polarity, not only include  $V_2$  and  $\frac{d}{1-d}V_2$ , but also include  $\frac{1-2d}{1-d}V_2$ , which results from  $V_1$  and  $V_2$  being stacked together. This voltage transfer ratio means this circuit can achieve the four quadrant operation. The recent family of Z-source dc/dc converter [5] can be categorized into this group. The classic SEPIC converter can also be included into this category, with the input equaling  $-V_2$  and the output equaling  $V_4$ .

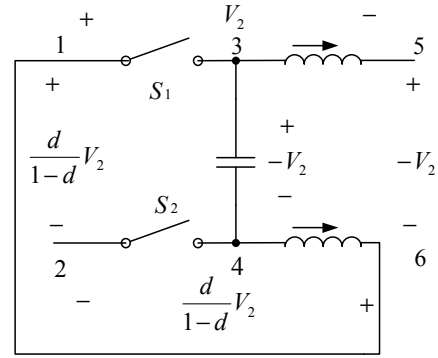


Fig. 6. The third order switching cell with  $V_2+V_3$  short circuited.

3. Points 2 and 5 are short circuited.

When points 2 and 5 are connected together, the sum of  $V_3$  and  $V_4$  equals 0.

$$V_3 - V_4 = 0 \Rightarrow V_3 = V_4 = -\frac{d}{1-d}V_2 \quad (15)$$

$$V_1 = \frac{1}{1-d}V_2 + V_3 = V_2 \quad (16)$$

Fig. 7 shows the voltage relationship among different voltage stiff elements. This case is essentially the same as the last case due to the symmetric topology of the third order switching cell and a total of six converters with three voltage transfer ratio could be derived.

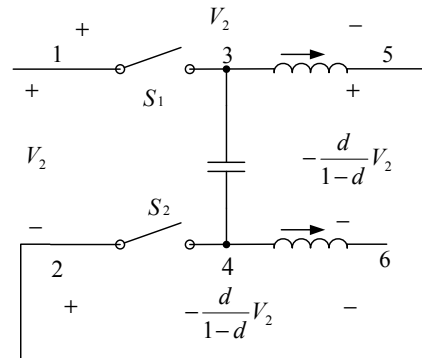


Fig. 7. The third order switching cell with  $V_3+V_4$  short circuited.

### B. Two inputs condition.

There are three groups of voltage stiff elements in the circuit and two of them are independent. So this circuit can be used as a two-input circuit. The circuit describing functions are (11), (12) and (13).

Fig. 8 shows one example in which the inputs are  $V_{in1} = V_1$  and  $V_{in2} = V_5$  and the output is  $V_{out} = V_2 + V_3$ . Using (11),(12) and (13), it is easy to find the relationship between output and input:

$$V_{out} = V_{in1} \times d + V_{in2} \times (1-d) \quad (17)$$

This topology could be used in a battery balancing circuit.

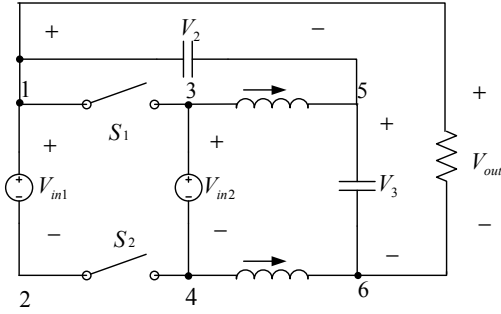


Fig. 8. The third order switching cell in two inputs mode.

## V. MULTI-INPUTS CONVERTER ANALYSIS

The multi-input converter (MIC) is a hot research topic in recent years which could find very broad applications in the synthesis of different renewable energy sources. MICs can usually be classified into two groups: with a transformer or transformer-less. This paper will focus on the transformer-less type and use the first order switching cell and third order switching cell to analyze the operation principle of transformer-less MICs.

The operation principle of transformer-less MICs is to add more switches into the circuit. Only one switch is closed at one time instant and the sum of the switching duty ratio of these switches is 1.

### A. First order case.

For the first order case shown in Fig. 9, there is only one inductor in the circuit and the following equation exists:

$$V_1 \times d_1 + V_2 \times d_2 + \dots + V_n \times d_n = 0 \quad (18)$$

from which the general equation for MIC circuit using first order switching cell can be derived.

$$V_1 = -\frac{1}{d_1} (V_2 \times d_2 + \dots + V_n \times d_n) \quad (19)$$

This circuit is proposed by Dobbs and Chapman in [6].

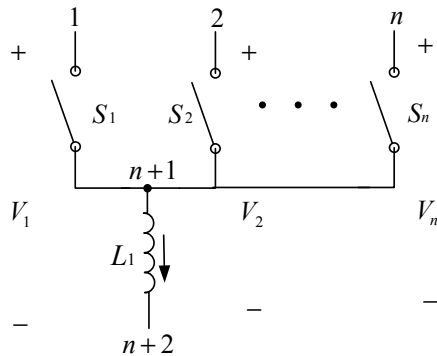


Fig. 9. The MIC derived from the first order switching cell.

### B. Third order case.

Another type of MICs could be derived from the third

order switching cell. In this case, the inductor/switch ratio is 1:1, more inductor-switch-capacitor branches will be added into the circuit. So although it is derived from a third order switching cell, it has an order higher than three. A two-input converter based on the third-order switching cell concept is shown in Fig. 10.

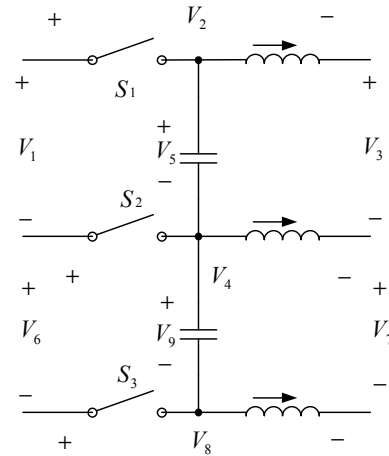


Fig. 10. The two-input MIC derived from the third order switching cell.

For each inductor, one equation exists, and by solving these equations, the following results are obtained:

$$V_2 \times d_1 + V_4 \times d_2 + V_8 \times d_3 = 0 \quad (20)$$

$$V_3 = V_5, V_9 = V_7 \quad (21)$$

Equation (20) is essentially the same as (18) and is the basis of third-order switching cell based MIC. Using the three type of connection method described in Section III, several MICs can be derived, two of them are proposed in [7] and [8] with the name of MICUK and MISEPIC converter.

## VI. CONCLUSION

This paper focused on the dc/dc converter topology and investigated a method of using stackable voltage stiff elements in the circuit in order to create new converter topologies. The first order switching cell is used and a third order switching cell is proposed and used to analyze first order and third order converters. New converters can be realized as possible alternatives for existing topologies. The application of this method in multi-input converters was also analyzed.

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