Weight Oriented Optimal PWM in Low Modulation Indexes for Multilevel Inverters with Unbalanced DC Sources

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***Abstract*—Selective harmonics elimination for the staircase voltage waveform generated by multilevel inverters has been widely studied for medium and high voltage applications. Most published strategies are for multilevel inverters with balanced dc levels and single switching per level. This paper proposes an Optimal Pulse Width Modulation (OPWM) method that utilizes equal area criteria and harmonic injection. In particular, for low modulation indexes, the weight oriented OPWM is proposed to determine switching angles to achieve accurate harmonic elimination.**

***Keywords*: Equal Area Criteria, Harmonics Injection, Optimal PWM, Weight Oriented**

I. INTRODUCTION

Because of the recent development of flexible ac transmission devices, medium voltage drives, and distributed generations, design and implementation of multilevel inverters circuits have been widely studied [1]-[3]. For these medium and high power applications, selective harmonic elimination based optimal pulse width modulation can be utilized to reduce the switching frequency while simultaneously keeping the total harmonics distortion (THD) under the numbers specified in relevant regulations and standards [4]-[6].

In 1964, the basic idea for selective harmonic elimination was proposed by Turnbull based on the Fourier series [4]. In this method, harmonic components are described as functions of the switching angles in trigonometric terms. Then a multiple variable equation group is formed, with one equation to guarantee the amplitude of the fundamental component, and the other equations ensuring elimination of the harmonics. If N is the total number of switching transitions, then, by calculating the switching angles, N-1 harmonics can be eliminated [6]-[10].

Some earlier algorithms use the Newton-Raphson method or linearization for real time calculation and harmonic elimination [11]-[12]. Some recent research on this topic utilizes advanced algorithms and control theories such as fuzzy logic solution, resultant theory, and sliding mode control [13]. In [14] and [15], online calculations of the switching angels are introduced. However, all these methods are still based on solving complex groups of equations. Therefore, when the number of switching transients is high, it is quite difficult or time consuming to solve with current computation methods [16], [17].

To eliminate harmonic components in multilevel inverters, four simple equations based on equal area criteria and the harmonic injection method have been proposed recently [7]. In [18], this method was proposed for optimal PWM switching angles on inverters. Then in [19], this method was utilized for unbalanced dc levels in multilevel topologies. The main problem associated with the basic four-equation method is the amplitude difference between the desired and resulted fundamental voltage.

To solve this problem, in [20] and [21], modulation indexes are categorized in two groups: low modulation indexes with an additional switching angle when an extra dc level is available; and high modulation indexes with an adjusted switching angle, where no extra voltage level is available, as shown in Figure 1.

In lower modulation indexes, fewer dc levels are utilized to synthesize the staircase waveform. Therefore, an extra voltage level is available and can be used for fundamental voltage compensation. Based on this idea, an additional switching angle in the extra voltage level is calculated to achieve desired fundamental voltage. For unbalanced dc sources, the following two equations are utilized:

- First, the fundamental voltage based on switching angles from θ_1 to θ_m is calculated with the following equation:

$$V_{1m} = \sum_{i=1}^{m} \frac{4V_{dc(i)}}{\pi} \cos(\theta_i), \qquad m < N \tag{1}$$

Then, an additional switching angle is calculated to

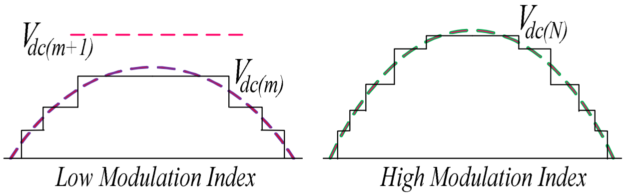


Fig. 1: Two groups of modulation indexes in multilevel inverters with unbalanced dc levels.

compensate for the difference between V_{1m} and fundamental voltage:

$$\theta_{m+1} = a\cos(\frac{\pi}{4V_{dc(m+1)}}(V_F - V_{1m})) \tag{2}$$

Thus, based on the four-equation based method, and through (1) and (2), harmonic components are eliminated by "m" switching angles and the reference fundamental voltage is compensated adequately by θ_{m+1}.

For the adjustment switching angle in higher modulation indexes, the fundamental voltage is corrected by the switching angle modification performed in the last DC level. This adjustment for unbalanced dc sources will be defined by the following equations:

- For fundamental voltage correction, an "adjustment" switching angle is calculated to by the following equation:

$$\theta_N^* = a\cos(\frac{\pi}{4V_{dc(N)}}(V_F - V_{1N})) \tag{3}$$

where V_{1N} is the total fundamental voltage generated by switching angles from θ_1 to θ_N.

- This "adjustment" angle is used to modify the switching angle for the last voltage level:

$$\theta_{N(\text{modified})} = a\cos(\cos(\theta_N) + \cos(\theta_N^*)) \tag{4}$$

Based on the switching angle adjustment for the last dc level, the desired voltage magnitude in the fundamental frequency can be achieved. However, the "adjustment" switching angle also causes new harmonic components used in the final modulation waveform in equal area criteria.

In this paper, the four-equation method is further developed for a multilevel inverter with unbalanced dc levels and multiple switching angles per level.

II. OPTIMAL PWM COMBINED WITH FOUR EQUATION BASED METHOD

In the basic four-equation based method in multilevel inverters, the harmonics selected for elimination are limited by the number of available dc levels. To overcome this problem, in each dc level, the number of switching angles can be increased to eliminate more harmonic components, as shown in Fig. 2. This is very helpful especially for low modulation indexes where limited dc levels are available.

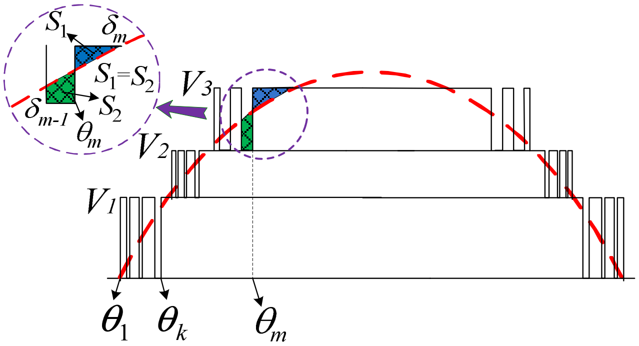


Fig. 2. Optimal PWM with four-equation based method on multilevel inverters with unbalanced dc sources.

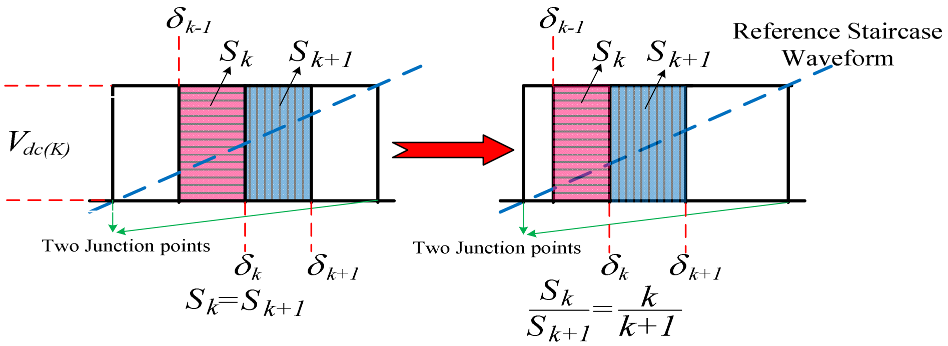
In theory, there is no limitation for the number of switching angles used for each level. Multiple carrier based PWM has been the most popular method. But the multiple carrier approach will not solve the issue with unbalanced dc levels. Furthermore, if the number of switching angles is limited by the switching loss, the multiple carrier PWM will not be able to realize effective harmonic elimination efficiently. So in this paper, the optimal PWM method is combined with the four-equation method to achieve selected harmonic elimination for multilevel inverters. In the basic four-equation method, switching angles are determined with equal area criteria by utilizing two adjacent junction points of the reference waveform and the dc voltage levels. When optimal PWM is used to increase the number of the angles for each level, switching angles cannot be found by just utilizing two major junction points. Thus, the area between the two junction points should be distributed in a reasonable pattern to achieve sub-junction points for switching angle calculation. With two sub-junction points, δ_{k-1} and δ_k, the switching angles are determined through the following equation:

$$\theta_k = 1/V_{dc(k)} * (\sum_{i=1}^{k} V_{dc(i)}\delta_k - \sum_{i=1}^{k-1} V_{dc(i)}\delta_{k-1} + V_F(\cos(\delta_k) - \cos(\delta_{k-1})) \\ - \frac{h_5}{5}(\cos(5\delta_k) - \cos(5\delta_{k-1}))\ldots - \frac{h_m}{m}(\cos(m\delta_k) - \cos(m\delta_{k-1}))) \tag{5}$$

For medium and high modulation indexes, in each voltage level, the available area can be equally divided for corresponded switching angles, as shown in Fig. 3 (a). However, for low modulation indexes, this strategy does not work well. This is simply because that with less voltage levels, larger "area" is needed to compensates selected harmonic components precisely. So the distribution of the δ_{k-1} and δ_k, becomes very crucial. Therefore, to overcome this problem in low modulation index, a weight oriented solution is proposed in this paper to decide the δ_{k-1} and δ_k.

III. WEIGHT ORIENTED SOLUTION

In this four-equation and OPWM combined method for multilevel inverters with unbalanced dc sources, the harmonics that are injected into the reference waveform are calculated by the following equations through Fourier series:



(a) Medium and high modulation index; (b) low modulation index

Fig. 3. Two methodologies for area division in OPWM and four-equation combined method.

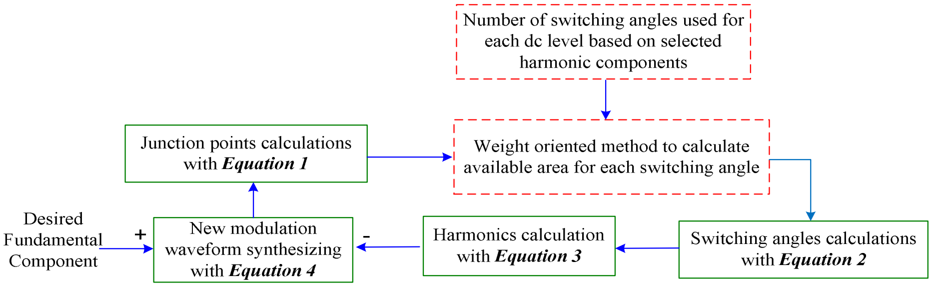


Fig. 4. Block diagram for weight oriented solution in low modulation indexes.

$$h_m = \sum_{k=1,2,..,N}^{N} \frac{4V_{dc(k)}}{m\pi}(\cos(m\theta_k) - \cos(m\delta_k)) \qquad (6)$$

It is clear that the magnitude of harmonic content decreases as the order of the harmonics increases. Therefore, the corresponded area necessary to eliminate higher order harmonics is decreased as well. Based on this observation, the area division for low modulation indexes is determined by the weight of the harmonics, which is shown in Fig. 3(b).

In this weight orientated solution, there is more area available for lower harmonic components with higher magnitude. Therefore, select harmonics can be eliminated in low modulation indexes with better accuracy. The procedure for this method is illustrated in Fig. 4.

If λ_k is defined by the difference between two sub-junction points, δ_k and δ_{k-1}; then the following equations can be used for λ_k:

$$\lambda_k = \delta_k - \delta_{k-1} \qquad (7)$$

$$\frac{\lambda_{k+1}}{\lambda_k} = \frac{k+1}{k} \qquad (8)$$

In this case, for a symmetric waveform, sub junction points are determined until $\pi/2$. Therefore for first level, if we assume that, switching angles are between 0 and $\pi/2$, then:

$$\pi/2 = \sum_{k=1}^{m} \lambda_k = \lambda_1 * \sum_{k=1}^{m} k = \lambda_1 * \frac{m*(m+1)}{2} \qquad (9)$$

So:

$$\lambda_1 = \frac{\pi}{m*(m+1)} \qquad (10)$$

Then, through (7) and (8), other sub-junction points can be determined easily.

A similar procedure can be used if the number of levels is more than one. In those cases, λ_k is calculated between two main junction points for each dc level. Thus, based on this procedure, switching angles are defined more effectively for lower modulation indexes. This idea is verified in simulations and experimental tests in next part.

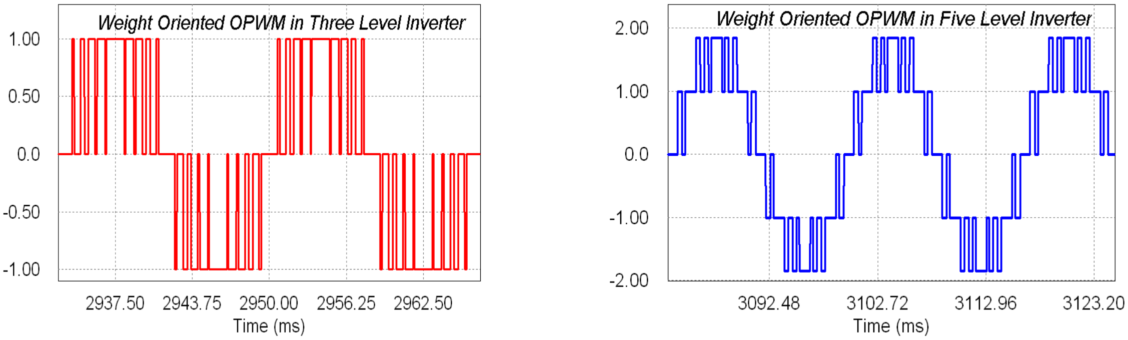
To test the proposed method, it is applied to two cascade multilevel inverter based cases: 1) one H-bridge and 2) two H-bridges with unbalanced dc levels. Five switching angles are used in both cases for harmonic elimination. The results in Table 1, show that five harmonic components can be eliminated effectively, thus verifying the advantage of the proposed method in low modulation indexes.

IV. SIMULATION RESULTS

To verify the switching angles shown in Table 1, a multilevel inverter with unbalanced dc levels is simulated for low modulation indexes. Table 2 shows the selected harmonic components for the simulated voltage waveform shown in Fig. 5(a) and (b). It can be seen that these harmonics are minimized successfully.

Table 1. Sample points based on the four-equation and OPWM combined method for low modulation indexes

# levels	Modulation Index	Switching Angles (rad.)					Harmonics (% based on fundamental output)					
		θ_1	θ_2	θ_3	θ_4	θ_5	1st	5th	7th	11th	13th	17th
1	0.1348	0.2438	0.5220	0.7486	1.0185	1.3050	100.00	0.00	0.8949	0.00	1.6620	1.1135
	0.1414	0.2450	0.5070	0.7461	1.0091	1.2887	100.00	0.00	0.5404	0.00	1.0035	1.1360
2	0.1901	0.3546	0.5497	1.0294	1.1829	1.3403	100.00	0.00	0.00	0.00	0.00	0.00
	0.2278	0.1159	0.4645	0.7975	1.2422	1.4205	100.00	0.00	0.00	0.00	0.00	0.00
	0.2545	0.1763	0.4578	0.8411	1.1378	1.3779	100.00	0.00	0.00	0.00	0.00	0.00



(a) Three level inverters

(b) Five level inverters with unbalanced DC sources

Fig. 5. Simulated voltage based on four-equation and OPWM combined method for multilevel inverters with unbalanced DC links.

Table 2. Simulation results for output voltage and selected harmonic components on different modulation indexes.

Modulation Index	Switching Angles (rad.)					Harmonics (% based on fundamental output)					
	θ_1	θ_2	θ_3	θ_4	θ_5	1st	5th	7th	11th	13th	17th
0.1414	0.2450	0.5070	0.7461	1.0091	1.2887	100.00	0.1532	0.5542	0.1673	0.9094	0.9700
0.2545	0.1763	0.4578	0.8411	1.1378	1.3779	100.00	1.1723	0.1256	0.8080	0.8237	0.3047

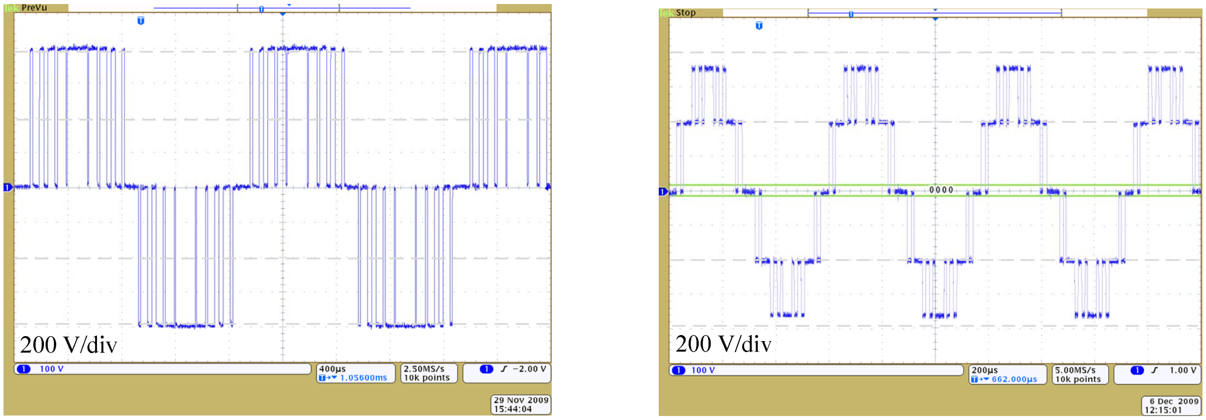
V. CASE STUDY

To verify the accuracy of the proposed method, 5 calculated switching angles, the same used for simulation, are applied to multilevel inverters. The experimental setup is shown in Fig. 6. In Fig. 7 (a) and (b), the output voltages for single inverter and a multilevel inverter with unbalanced dc voltage are shown respectively. In the unbalanced case, the second dc level is 80% of first level. From the harmonics analysis in Table 3, it can be seen that the weight oriented method can be used successfully in low modulation indexes.

VI. CONCLUSION

Most published methods for harmonic elimination in multilevel inverters are proposed for balanced dc sources with a single switching angle for each dc level. In this paper, a modified four-equation method is proposed for unbalanced dc levels with multiple switching angles per level. A weight oriented solution is used to identify the junction points needed in the four-equation method for low modulation indexes. Simulation and experimental results are shown to verify the proposed method.

Fig. 6. Experimental setup for multilevel inverter with unbalanced dc sources.



(a) Three level inverters

(b) Five level inverters with unbalanced DC sources

Fig. 7. Experimental results for single and multilevel inverter with unbalanced dc sources.

Table 7. Selected harmonics analysis for two case studies.

Modulation Index	Switching Angles (rad.)					Harmonics (% based on fundamental output)					
	θ_1	θ_2	θ_3	θ_4	θ_5	1st	5th	7th	11th	13th	17th
0.1414	0.2450	0.5070	0.7461	1.0091	1.2887	100.00	2.2365	1.8975	1.2392	0.8451	0.3652
0.2545	0.1763	0.4578	0.8411	1.1378	1.3779	100.00	3.0636	1.4018	0.8134	0.7811	0.5384

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