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A Novel Converter Fed Reluctance Motor With High Power Density

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A NOVEL CONVERTER FED RELUCTANCE MOTOR WITH HIGH POWER DENSITY

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Abstract This paper introduces a new type of synchronous reluctance motor having only two phases. Since the currents in the phases are unidirectional rather than bi-directional, the associated power converter requires only two transistors and two feedback diodes. It is demonstrated that with the same amount of active copper, this machine is capable of twice the torque as the equivalent switched reluctance motor.

INTRODUCTION

Although induction motor drives are still the workhorse of industry, the switched reluctance (SR) motor drive has been actively researched over the past decade with very promising results [1-3]. The SR machine has a simple and rugged construction and very good overall performance over a wide torque-speed range [2]. Most importantly, this machine requires only unipolar current for excitation since the rotor is not polarized. Hence, simpler power converter structures to power the machine are possible. A detailed comparison of the SR motor drive with a high efficiency induction motor drive indicates that performance parameters, such as torque per unit stator volume, torque per unit inertia and torque per unit copper weight, can be made equal to that of induction machine, or, in some cases, even exceed the induction machine [3].

The SR motor is, in effect, an advanced type of stepping motor in which the current is carefully regulated to provide maximum torque per "stroke" of phase current. The designation "switched reluctance" is a somewhat inappropriate term for this machine since induction and synchronous motors operated from an inverter are never termed "switched induction" or "switched synchronous" motors. In this paper, the term current regulated stepping motor (CRS motor or CRSM) is used as a more appropriate and consistent term for this machine when compared with other conventional ac motor drives operated with a closed loop current regulator.

While the recent work on CRS machines are encouraging, the jury is still out on whether the machine is truly an optimum geometry. In several respects the machine is suspect. For example, taking a 8 stator/6 rotor pole CRS motor as an illustration, only one quarter or, at best, with excitation of two of the four phases, one half of the stator inner circumference is utilized to make a contribution to torque development at any instant. Secondly, the inductance variation or the occupying coils over each one fourth of the machine is limited by the so called double salient design. Hence, it is clear that any method of ensuring that the other three quarters of the inner circumference of the stator remain "active" will be very significant to torque production and, therefore, to increasing the power density of the machine. The same argument applies if the inductance variation per unit of rotor rotation can be increased by other methods.

This paper describes a new configuration of a synchronous reluctance motor utilizing concentrated windings and unidirectional winding currents which shows promise in outperforming the CRS motor. The paper proposes that with an identical frame size and same amount of active copper, the output of the machine can be doubled compared to its CRS motor counterpart. The striking improvement of feasible torque production in a synchronous reluctance motor is attributed to the multi-circuits in each phase which ensures that the entire airgap surface remains "active" and also to a segmented rotor construction which allows the variation of the inductance of a coil to be several times of that of CRS motor. Since this machine also requires careful regulation of the stator current to extract maximum torque, the machine is denoted in this paper as a current regulated synchronous reluctance motor or CRSM.
Based on the concepts presented in this paper, a suitable number of stator phases and rotor poles are chosen for both CRS and CRSR motors with the same frame size and torque production is analyzed and compared. The detailed design and fabrication of such a prototype CRSR motor is in progress and the test results will be presented in the near future.

REVIEW OF TORQUE PRODUCTION IN AN CRS MOTOR

Analysis of Basic CRS Motor Operation

Electromechanical energy conversion in a CRS motor, shown idealized in Fig. 1, is accomplished by means of a time varying inductance due to the variation of the rotor position. This principle can be best illustrated by plotting a stator winding inductance profile versus the rotor position with respect to the axis of the stator winding and the stator current waveform corresponding to the variation of this inductance as shown in Fig. 2. It should be noted that the current is idealized and assumed to flow only during the interval when motoring torque production is possible.

Fig. 1 Idealized Representation of Eight Pole Stator - Six Pole Rotor Switched Reluctance (Current Regulated Stepping) Motor.

The physical operation of the model can also be represented by the simple circuit shown in Fig. 3. For simplicity of analysis, all the losses are neglected for the time being, and therefore no resistance is involved in the circuit. The circuit is assumed to be fed with a prescribed current as shown in Fig. 2 from the source and the inductance is time-varying due to the rotor rotation. The flux linkage associated with the inductance L is \( \lambda \) and if saturation of the magnetic material is neglected, the flux linkage can be expressed as

\[
\lambda = L(\theta_r) i
\]

where \( L \) is only a function of rotor angle displacement (saturation neglected). The induced voltage by the inductance is

\[
E = \frac{d\lambda}{dt} = L \frac{di}{dt} + \frac{i}{L} \frac{dL}{d\theta_r} \frac{d\theta_r}{dt}
\]

The instantaneous power entering the circuit is

\[
P = Li \frac{di}{dt} + i^2 \frac{dL}{d\theta_r} \frac{d\theta_r}{dt}
\]

where the rotor angular speed \( d\theta_r/dt \) is assumed to be a constant, \( \omega \). Equation (3) can be further written in the form of

\[
P = \frac{1}{2} L \frac{d^2 i}{dt^2} \frac{d\theta_r}{dt} + \frac{i}{2} \frac{dL}{d\theta_r} \frac{d\theta_r}{dt} + \frac{1}{2} i^2 \frac{dL}{d\theta_r} \frac{d\theta_r}{dt} - \frac{1}{2} i \frac{d^2 L}{d\theta_r^2} \frac{d\theta_r}{dt}
\]
\[
\frac{d}{dt} \left[ \frac{1}{2} L i^2 \right] + \frac{1}{2} i^2 \frac{dl}{d\theta} \omega = 0
\]

Equation (4) yields the well known result that the input electrical power is balanced by the derivative of the stored field energy and the mechanical output power. The second term in the right hand side of (4) indicates that the electromagnetic torque can be expressed as:

\[
T = \frac{1}{2} i^2 \frac{dl}{d\theta}
\]

Limitation of Improvement in Torque Production in CRS Motors

By examining (5), the following conclusions can be reached immediately:

1) Motoring torque is produced if the CRS machine is excited during the interval in which the inductance of the winding is increasing, that is, when \( dl/d\theta \) is positive.

2) Generating torque is produced if CRS machine is excited during the interval in which the inductance of the winding is decreasing, that is, when \( dl/d\theta \) is negative.

3) Torque production is proportional to the square of the current and therefore independent of current polarity if mutual inductance is not involved. That is, when only one winding is excited.

4) For a given current, to maximize the torque, \( dl/d\theta \) should be maximized.

From a motor design point of view, 4) implies that within a certain angle of rotor rotation, the stator winding inductance variation should be as large as possible. The limitation imposed by a CRS motor in maximizing \( dl/d\theta \) can be explained by taking a two pole rotor and one phase stator coil CRS motor as an example, as illustrated in Fig. 4. In order to maximize \( L_{\text{max}} \), the stator pole should be completely aligned with the rotor pole. On the other hand, to minimize \( L_{\text{min}} \) the stator pole should be totally nonaligned with the rotor pole. Therefore, a design constraint exists wherein the relationship

\[
\beta_r + \beta_s \leq 2\pi/N_r
\]

must be satisfied. Under this constraint, \( L_{\text{min}} \) is limited by the leakage flux and \( L_{\text{max}} \) is limited by the permeance of main flux path which is set by the cross-section of the stator pole and minimum allowable air gap. In general, these limitations generally result in a maximum achievable ratio of \( L_{\text{max}}/L_{\text{min}} \) of 10:1. By observing the illustration,
segment consists of a stack of axially laminated iron sheets sandwiched with nonmagnetic material. The rotor is fitted into a stator having fully pitched windings. To distribute the load the machine can be designed with any number of phases. However, a two phase CRSR machine most analogous to the 8/6 CRS motor. The phases are designed to carry unidirectional currents in the same manner as the CRS machine.

Several converter arrangements are possible to power this machine [1, 2, 6]. One of the simplest, using a split dc bus and only two transistors, is shown in Fig. 6(a). The currents are, in effect, unidirectional blocks of current with a duration of 90° electrical and occur with two pulses per cycle as shown in Fig. 6(b). Since the flux in each coil is a pulsing dc quantity, the rotor poles encounter an ac flux variation during rotation. Because the rotor poles are not polarized, a continuous unidirectional torque can still be maintained and, excluding saturation effects, the torque is a non-pulsating quantity.

Computation of Inductance as a Function of Rotor Position

It is convenient to analyze this machine by the "rotor induced MMF" method as was described in [4]. If the stator MMF is as shown in 7(a), then it can be shown that the induced rotor MMF will be as given in Fig. 7(b). It is interesting to note that the induced rotor MMF depends not only on the waveform of stator MMF but also on the position of the rotor with respect to the stator winding. As an extreme case, if 0 equals 90°, the induced MMF will have an identical waveform to that of the stator with opposite polarity which is extremely useful in the sense of maximizing $dL/d\theta$.

Fig. 7 Stator MMF and Induced Rotor MMF of CRSR Motor.

If we consider that the airgap flux is produced by the effective MMF which is the summation of stator MMF and rotor induced MMF, and if the airgap length is uniform, then winding function theory can be very conveniently employed to compute the inductance [7]. According to winding function theory, the inductance for a given coil can be calculated as

$$L_{ab} = \frac{\mu_0 H^2}{g^2} \int_{0}^{2\pi} F_{ab}(\theta) d\theta$$  \hspace{1cm} (7)

where $F_{ab}(\theta, \phi)$ is the effective MMF distribution created by a unit current in a given stator winding and $N_{a}(\theta)$ is the winding function expressing the stator coil distribution as a function of the angular measure along the air gap $\theta$. In the more general situation, the stator may be equipped with multi-phase windings. In this case the approach described above can be generalized to compute all self and mutual inductances.

Figure 8 shows the results of an inductance calculation for the six pole rotor with two phase stator windings mutually displaced by 90°. The rotor position $\theta_r$ with respect to phase winding A is 90° electrical for the case shown and the rotor is
assumed to move in the direction of increasing $\theta_r$.

The self inductances $L_{ss}$ and $L_{sh}$ calculated by
winding functions when the rotor moves from 0
to 120° are shown. It should be observed that
because the restriction $\beta_s + \beta_r \leq 2\pi/N_s$ is not applica-
cable for this structure, the iron cross-section of
the coil can be many times of that in double-
salienty design, the $L_{\text{max}}$ value in each coil will
clearly be many times of that of the CRS motor
winding.

It should be noted that this computation is
idealized in two respects. First, when the rotor is
at the extreme position ($\theta_r = 30^\circ$), it is assumed
that there is no leakage flux at all. This idealization
results in $L_{\text{min}}$ being zero. Secondly, it is
assumed that the magnetic material works in the
linear region only and that the reluctance in the
iron is zero. Hence, $L(\theta_r)$ is only a function of
rotor angle. In reality, these assumptions are, of
course, not true. Nevertheless, these idealizations
will not affect the general conclusion since it is the
difference between $L_{\text{max}}$ and $L_{\text{min}}$ that contributes
to torque production.

**Comparison of Torque Capability of CRS and CRSR Motors**

Prototype CRS and CRSR motors are shown
in Figs. 1 and 5. For the purpose of compari-
son, the following assumptions are made:

1) The two motors have the same stator inner
diameter, the same length of the iron stack
and the same length of airgap.

2) The number of rotor poles are the same for
both motors.

3) Both motors have the same total number of
turns of stator windings and the wire gauge is
the same and therefore the two machines
have the same amount of active copper
weight.

4) Each stator phase winding has the same
current and because of 3), the same current
density.

5) The CRSM has 8 stator coils and CRSR
motor 12.

6) The magnetic materials in both motors work
in the linear region.

7) Consistent with manufacturing experience,
the ratio $L_{\text{max}}/L_{\text{min}}$ is assumed to be 10 and 5
for the CRS motor and CRSR motor respec-
tively.

Based on the assumptions 1) through 6) it
immediately follows that the turns per coil and
MMF produced by a coil in a two phase, six pole
CRSR motor is 2/3 of that in a four phase, eight
pole CRS motor. The flux density in a CRSR
motor is thus 2/3 of that in a CRS motor.
Because both motors have the same number of
rotor poles, each coil in both the CRS and CRSR
motors is excited six times for each revolution of
the rotor. The period for excitation of each coil in
a CRS motor is 15 degrees. However, for a coil in
a CRSR motor it is 30 degrees.

It can be recalled that the instantaneous
torque during the period of excitation of any coil
is governed by Eq. (5). For convenience, it will be
assumed that during the excitation period the coil
inductance is a linear function of rotor position $\theta_r$.
Then (5) can be written in the form:

$$ T = \frac{1}{2} i^2 \frac{L_{\text{max}} - L_{\text{min}}}{\Delta \theta} $$

where $L_{\text{max}}$ and $L_{\text{min}}$ are the inductances in the
maximum and minimum inductance positions
respectively and $\Delta \theta$ is the incremental rotor angle
over which this inductance variation takes place.

It is convenient to let

$$ k_L = 1 - \frac{1}{L_{\text{max}}/L_{\text{min}}} $$

The torque can be then be written as

$$ T = \frac{1}{2} i^2 k_L \frac{L_{\text{max}}}{\Delta \theta} $$

It can be recalled that the inductance can be
expressed as [7]

$$ L_{\text{max}} = \frac{\mu_{\text{o}} / 2 \theta}{\mu_{\text{o}} / 2 \theta} $$

where

$\mu_{\text{o}}$ = permeability of air

l = length of the stack
$r$ = radius of the rotor
$\theta_0$ = pole arc of one stator pole
g = length of airgap
$N$ = # of turns per coil

Substituting (10) into (9), the average torque contributed by a single coil in one rotor revolution is therefore,

$$T_{(ave)} = \frac{6}{2\pi} \left[ \frac{1}{2} r^2 \frac{m \mu_0 l_r \theta_0 N^2}{2g} \right]$$ (11)

It is important to note the difference between the instantaneous torque and the average torque which is independent of the incremental rotor angle $\Delta \theta_r$. Taking the number of phases and number of circuits per phase into account, the total average torque produced by the machine is

$$T_{(ave)} = mC \left[ \frac{1.5}{\pi} r^2 \frac{k_l \mu_0 l_r \theta_0 N^2}{2g} \right]$$ (12)

where

$m$ = number of phases
$C$ = number of circuits per phase

For purpose of comparison, it is convenient to simply denote all quantities associated with CRSR motor by the subscript "1" and those of the CRSR motor by "2". Therefore, a torque ratio expressing the degree of improvement of the CRSR motor relative to the CRS motor can be written as:

$$\tau = \frac{T_{(ave)2}}{T_{(ave)1}} = \frac{m_2 C_2 k_{l2} \theta_{02} N^2}{m_1 C_1 k_{l1} \theta_{01} N^2}$$ (13)

where we have already made use of the fact that $i_1 = i_2$. This expression clearly indicates the importance of the iron cross sectional area spanned the coil i.e., the importance of $\theta_0$. From assumptions 1) through 7), we have

$N_2 = 0.67 \times 10^3 \quad N_1 = 1 \times 10^3$

$m_2 = 2 \quad m_1 = 4$

$C_2 = 3 \quad C_1 = 1$

$k_{l2} = 4/5 \quad k_{l1} = 2/10$

$\theta_{02} = 60^\circ \quad \theta_{01} = 18^\circ$

Upon evaluation of (13) using the above parameters, the torque ratio between the CRSR motor and CRS motor becomes

$$\tau = 1.997$$ (14)

Hence, with the identical frame size and copper weight, the CRSR motor will develop twice as much torque as that of the CRS motor. That is, the power density of a CRSR motor will be twice that of a CRS motor. This conclusion may also be understood by inspection of the CRSR motor and CRS motor operation physically. Note in particular that during each energy conversion period, the CRS motor only has one coil making a contribution to torque production while the CRSR motor results in the entire airgap surface being active. In addition, each coil in the CRSR motor links several times more flux than the coil in the CRS motor due to the segmented rotor structure. Therefore, it is not surprising that CRSR motor can convert more energy for the same rotor speed.

**FREQUENCY AND WAVESHAPE OF THE FLUX VARIATION**

Although the CRSR motor develops as much as two times the torque of a CRS motor with the identical physical size, the flux in the motor for different moments of operation still retains desirable characteristics. In particular, Fig. 9 depicts the airgap flux distribution for $\theta = 0^\circ$, 7.5°, 15°, 22.5° and 30°. Because there is no current overlap between the phases, the flux at any moment is produced by a single phase current only. The flux distribution around the airgap has the following features,

1) The total flux linkages increase linearly with the rotor position from 0 to 30 degrees, but the magnitude of the flux density at any point is always 2/3 that of the equivalent CRS motor.

![Fig. 9 Air Gap Flux Distributions for Various Rotor Positions.](image-url)
2) The fundamental frequency variation of the flux pattern is reduced to half of that in the CRS motor.

In summary, while the flux density in CRSR motor is reduced to 2/3 of that in CRS motor the increased span of the coils keeps torque production high. It is clear that this mode of operation has important side benefits since the iron losses in CRSR motor would be significantly reduced due to a reduction in both the frequency and magnitude of the flux density. Also, it is clear that saturation will affect the torque production capability of both machines. However, since the flux density is 33% lower in the CRSRM than the CRS motor, saturation should only serve to improve the situation in favor of the new machine.

It should be mentioned, however, that the copper losses in the new machine will increase because more coils are conducting during a longer portion of each excitation interval. However, an overall efficiency improvement for the CRSR motor is expected. Other advantages of operating the motor with low frequency flux variation are:

1) High speed capability due to the low ratio of switching frequency to speed.
2) Less number of switches in power converters.
3) Reduction of losses associated with the switching frequency.
4) Since the flux density in the rotor is non-alternating, the rotor can be constructed with grain-oriented steel further reducing the iron losses.

CONCLUSION

Although switched reluctance (stepping) motors have attracted considerable interest as a potential replacement for induction motors, the similar capabilities of more conventional singly salient synchronous reluctance motors have been neglected. This paper shows that the desirable features of unidirectional current, low torque pulsation, and low losses can also be accomplished with a synchronous reluctance motor and with a marked improvement in the power density. While any number of phases can be employed in this new machine, the special advantages of a two-phase machine were outlined. While the results are theoretical thus far, a prototype machine of the required geometry has been designed and will soon be tested in our laboratory.

REFERENCES