Adaptive Fuzzy Control of a Variable Speed Power Generating System with Doubly Excited Reluctance Machine

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Abstract- Fuzzy control methods are developed for a high efficient variable speed constant frequency power generating system, with wind power generation as an example. In slip power recovery configuration, the system is composed of a doubly excited brushless reluctance machine and a reduced rating power electronic converter unit linking the stator secondary circuit with the power line. The doubly excited brushless reluctance machine has high efficiency, high reliability and rugged construction. Under fuzzy logic control which enables superior dynamic performance, the operational speed is controlled to track the optimal power-speed profile of the wind turbine. Reactive power is independantly controllable. Active and reactive power capacities of the system are also discussed.

I. Introduction

Slip power recovery systems composed of a wound-rotor induction machine and power electronic converters in the rotor circuit can be used as variable speed drives and generators. Comparing to conventional AC machine systems, the advantages of slip power recovery systems (SPRS) include higher efficiency and lower power converter rating. However, the wound-rotor structure requires rotor slip-rings while bringing additional losses in the rotor circuit.

The Doubly Excited Brushless Reluctance Machine (DEBRM) has two sets of stator windings, and no windings in the rotor which has simple saliency or is axially laminated [1]. Like a squirrel cage induction machine, no brush or slip-ring is needed, offerring a rugged construction. Unlike a squirrel cage induction machine, there is no short circuit rotor current. In addition, with a slip power recovery configuration, energy efficiency can be further improved. However, high performance control of the SPRS has been difficult, with either a DEBRM or a conventional wound-rotor induction machine [2].

Wind power generation is one of the alternatives of electricity generation in which variable speed constant frequency (VSCF) operation provides an impetus for developments and acceptance. In the advanced VSCF wind power generating system, control objectives are: 1) To track a prescribed power-speed curve, for maximum energy capturing of the wind turbine; 2) The constant output voltage has a constant frequency; 3) Flexible reactive power control is achievable; 4) High performance with good dynamic response. Such stand-alone or auxiliary power generating system provides an alternative in producing electricity. To be fully integrated into the bulk power system, more control objectives are introduced for the wind power generator in short range and long range generation scheduling [4].

If the turbine blades have been mechanically optimized, for each wind speed there is a cubic power-speed curve that relates captured power with turbine speeds. At a given wind speed, there is a corresponding turbine speed at which the wind turbine captures maximum power, whereas if running at a different speed, the captured power can be substantially less. More or less, the wind speed varies constantly, thus to maximize power captured from the wind, the turbine speed should also vary accordingly along the maximum power profile.

Constant tracking of the power-speed profile is a difficult task, if high dynamic performance is desired (such as no speed overshoot and no oscillations). With PID type control, the controller gains can be tuned to work best only for a small operation range. Nonlinearity and uncertainty of the problem indicate the possibility and challenging of using adaptive control techniques, which can provide good closed-loop performance even in the face of changing parameters and disturbances. However, for the complex doubly-excited machine and converter system, designing of a conventional adaptive controller can be difficult. Without using a mathematical model, fuzzy control provides a solution.

In this paper, a direct fuzzy logic controller is first designed for the generating system, followed by an adaptive fuzzy controller based on model reference adaptive control. Computer simulation results are presented. Active and reactive power capacities of the system are also discussed.

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II. System Structure and Model

The stator of the DEBRM is wound in dual sets of three phase windings, while the rotor consists of simple saliency or axially laminated segments without windings, brushes and slip-rings [1]. As shown in Fig. 1, one of the two sets of stator windings, the "primary windings", is directly connected to the constant frequency power line, while the other set, the "secondary windings", is connected to the power line via a power converter.

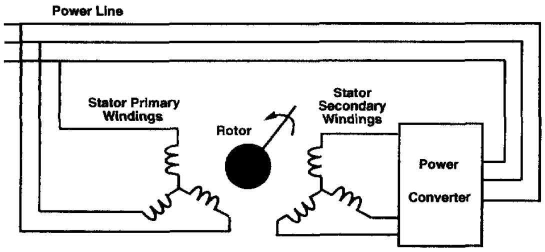

Fig.1 DEBRM Interconnection

If the pole numbers of the two sets of windings are "$2p_1$" and "$2p_2$" respectively, the rotor pole pair number is constrained to "p_1+p_2". To produce useful average torque, the primary frequency ω_1, the secondary frequency ω_2 and the rotor frequency ω_r should satisfy the following constraint:

$$\omega_1+\omega_2=\omega_r(p_1+p_2) \tag{1}$$

when the sequence of the three-phase currents in the two sets of stator windings are the same. Otherwise,

$$\omega_1-\omega_2=\omega_r(p_1+p_2) \tag{2}$$

when the sequence of the three-phase currents in the two sets of stator windings are opposite to each other.

Variable speed generator operation is realizable through (1) or (2). For example, given the primary frequency ω_1 to be a constant, the secondary frequency ω_2 can be controlled to match the rotor speed variation, satisfing (1) or (2). Similar to a typical slip power recovery system with a wound-rotor induction machine, the power converter draws slip-power from the secondary windings when the rotor speed is above the synchronous speed, and provides slip-power to the secondary windings when the rotor speed is below the synchronous speed.

It is advantageous to select the stator primary winding flux as the dq dynamic reference of all the machine quantities. In short, $\omega=\omega_e$ and $\lambda_{q1}=0$. In such a reference frame, the machine dynamical equations can be written as [2]

$$v_{d1}=r_1 i_{d1}+\frac{d\lambda_{d1}}{dt} \tag{3}$$

$$v_{q1}=r_1 i_{q1}+\omega_e\lambda_{d1} \tag{4}$$

$$\lambda_{d1}=L_1 i_{d1}+L_m i_{d2} \tag{5}$$

$$\lambda_{q1}=0=L_1 i_{q1}+L_m i_{q2} \tag{6}$$

$$v_{q1}^2+v_{d1}^2=v_m^2 \tag{7}$$

Since the d-axis of the reference frame is the instant axis of the primary winding flux, the phase angle of the line voltage is generally not a constant in the reference frame, although its frequency and magnitude are constants. The electromagnetic torque can be derived as

$$T_e=-\frac{3}{2}\frac{P}{2}\frac{L_m}{L_1}\lambda_{d1}i_{q2} \tag{8}$$

The level of the primary winding flux remains approximately unchanged, restricted by the constant magnitude and frequency of the line voltage. Therefore, as can be observed from (8), the torque or active power control can be achieved by controlling the secondary current component orthogonal to the primary flux.

The reactive power at the terminal of the primary winding can be derived as

$$Q_1=\frac{3}{2}(v_{q1}i_{d1}-v_{d1}i_{q1}) \tag{9}$$

or, from (3) and (4), with the primary flux remains unchanged,

$$Q_1=\frac{3}{2}\frac{P}{2}\omega_e\lambda_{d1}i_{d1} \tag{10}$$

As (5) indicates, i_{d1} is controllable by i_{d2}, with λ_{d1} unchanged. Therefore, the d-axis component of the secondary current, i_{d2}, can be controlled to regulate the primary winding reactive power.

As a result, the control of the electromagnetic torque T_e via i_{q2} and the control of the primary reactive power Q_1 via i_{d2} are essentially decoupled. The flux control is generally unnecessary since it maintains a constant level, while the flexible control of reactive power becomes possible. Other than supplying the required amount of reactive power to the power system, unity power factor operation can be maintained, with the machine coppor losses minimized [2].

III. Direct Fuzzy Logic Control

Before the adaptive fuzzy controller is introduced, a direct fuzzy logic controller is designed and simulated first, providing an immediate alternative to the field orientation control [2]. The generating system with direct fuzzy logic controller is shown in Fig. 2. Its major components are a DEBRM, dual PWM converters, a dc link, a reactive power regulator and a fuzzy logic controller (FLC). The components of the FLC are shown in Fig. 3.

For the FLC, the input valuables are the speed error, change of the speed error, and the output is the q-axis secondary current increment, for which the

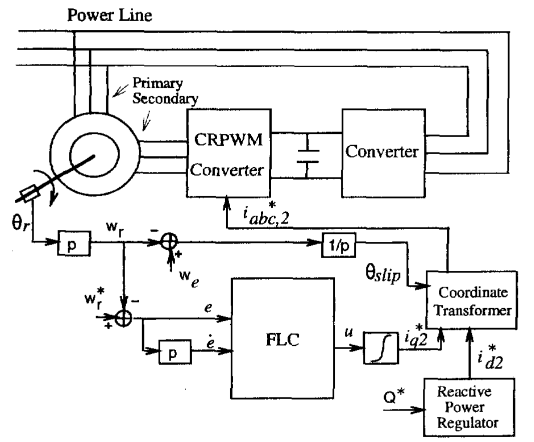

Fig.2 Generating System with Direct Fuzzy Control

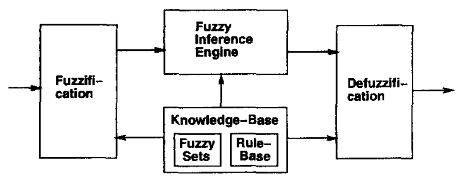

Fig.3 Fuzzy System

Table 1. Fuzzy Rules

u	E_2^1	E_2^2	E_2^3	E_2^4	E_2^5	E_2^6	E_2^7
E_1^1	7	7	7	6	6	5	4
E_1^2	7	7	6	6	5	4	3
E_1^3	7	6	6	5	4	3	2
E_1^4	6	6	5	4	3	2	2
E_1^5	6	5	4	3	2	2	1
E_1^6	5	4	3	2	2	1	1
E_1^7	4	3	2	2	1	1	1

fuzzy sets are denoted as E_1^j, E_2^j and U^j respectively, with $j = 1, ..., 7$. The linguistic values, in the order from 1 to 7, are NL(negative large), NM(negative medium), NS(negative small), ZE(zero), PS(positive small), PM(positive medium), PL(positive large). The knowledge base contains information on the fuzzy sets and a rule base with a set of linguistic conditional statements, as shown in Table 1.

A membership function $\mu_{A_i^j}(a_i)$ associates each member a_i with its grade of membership in the fuzzy set A_i^j. Triangular membership functions as shown in Fig. 4 and Fig. 5 are used for the input and output fuzzy sets, respectively.

The fuzzification process, a Singleton one, interprets an input a_0 as a fuzzy set with the membership function $\mu_A(a)$ equal to zero except at the point a_0, where $\mu_A(a_0)$ equals one. The fuzzy inference engine maps in-

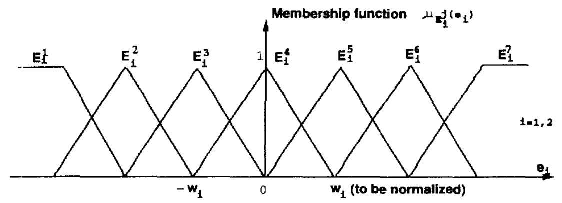

Fig.4 Membership Functions for Inputs

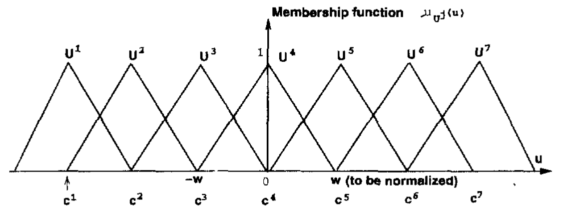

Fig.5 Membership Functions for Output

put fuzzy sets to output fuzzy sets based on the fuzzy IF-THEN rules and the compositional rule of inference, where the Sup-Product inference method is used for the premises of the fuzzy rules and the Sup-Min inference method is used for the fuzzy implications [3]. Note that

-Product $\mu_{A\star A'}(a) = \mu_A(a)\mu_{A'}(a), a \in \mathcal{A}$
-Min $\mu_{A\star A'}(a) = min\{\mu_A(a), \mu_{A'}(a) : a \in \mathcal{A}\}$

The defuzzification process maps the output fuzzy sets into a crisp output value. With the Center of Gravity defuzzification method, the output is calculated by

$$u^* = \frac{\sum_{j=1}^{7} c^j \int \mu_{\hat{U}^j}(u)du}{\sum_{j=1}^{7} \int \mu_{\hat{U}^j}(u)du} \tag{11}$$

where $\hat{U}^j$ is the implied fuzzy set after inferences and c^j is the crisp value at which the membership function $\mu_{U^j}(u)$ reaches its maximum, i.e. the center of each triangular membership function.

The inputs and the output are all normalized with tuning. Performance specifications can be met by adjusting the normalizing gains, with considerations of other limiting factors related with the machine and power converters, such as torque limit, current limits, etc.

The fuzzy logic controller is PD-like, i.e. it resembles the conventional PD controller in the form of

$$u = K_P e + K_D \dot{e} \tag{12}$$

where K_P and K_D are the proportional and the differential gains, respectively.

Simulation is conducted for a 50hp DEBRM with a 4-pole rotor. Fig. 6(a,b) show the step speed tracking dynamics of the system. Fig. 7(a,b) show corresponding q-axis and d-axis secondary currents. Note that the d-axis secondary current is controlled separately to maintain a specific amount of stator primary reactive power flow, such that the machine coppor losses is minimized [2].

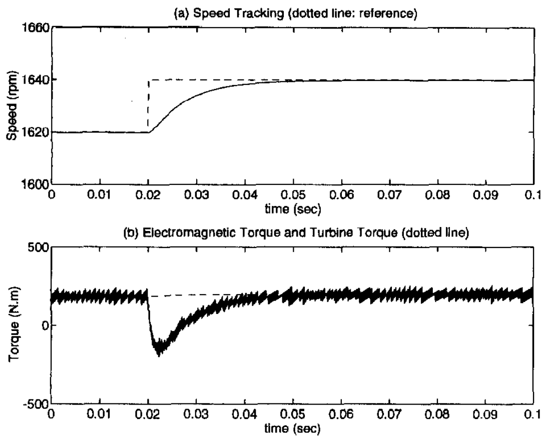

Fig.6 Direct Fuzzy Control Simulation

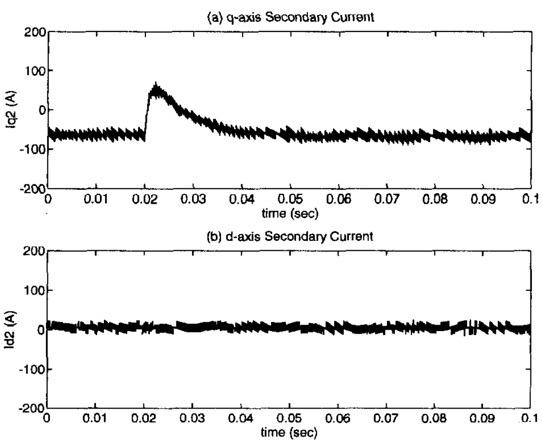

Fig.7 PWM Regulated Secondary Currents

Instantaneous speed tracking is simulated in Fig. 8. Note that the speed reference is not known in advance. In reality, wind speed changes much more slowly than simulated; therefore, the acceleration or deccelaration torques would be much smaller in speed transients.

Furthermore, rejection of parameter variations is achieved, as simulated in Fig. 9(a,b) when both the stator and rotor resistances change significantly and continuously. Similarily, disturbances can also be tolerated.

IV. Adaptive Fuzzy Logic Control

Fig. 10 shows the generating system with the adaptive fuzzy controller, which is composed of the direct

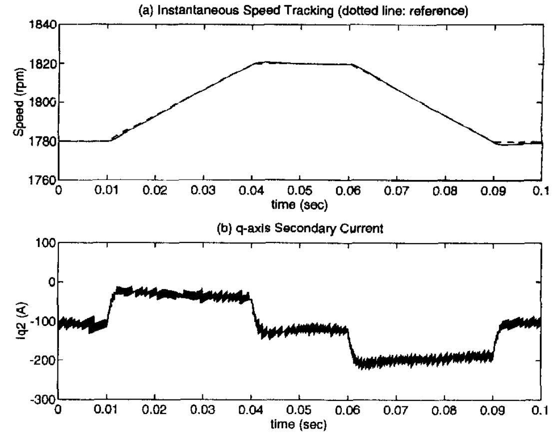

Fig.8 Instantaneous Speed Tracking

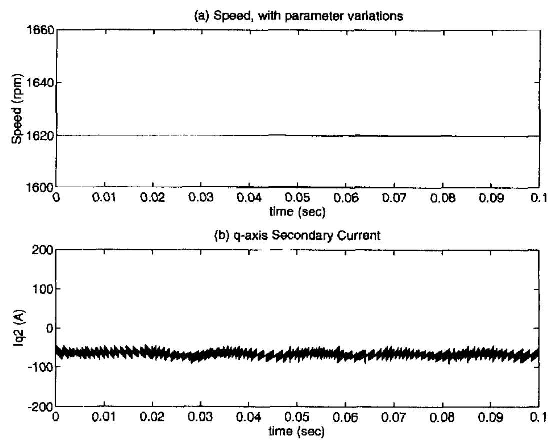

Fig.9 Simulation with Parameter Variations

fuzzy logic controller, a reference model, a fuzzy learner and an adaptor.

The principle of model reference adaptive control is employed in the system. The performance specifications are stored in the reference model, which uses the speed command, ω_r^*, to produce a reference response ω_r^{ref} that meets the desired performance specifications. Note that the performance specifications, including speed overshoot, rise time, settling time, etc, should be reasonable so that machine capabilities are considered. The reference response ω_r^{ref} is compared with the actual speed ω_r, and the error e^{ref} and change of error $\dot{e}^{ref}$ are the inputs to the fuzzy learner, which outputs the instruction m to adapt the direct fuzzy logic controller.

The design of the fuzzy learner is very similar to that of the FLC in the previous system, with the same fuzzy sets, rule base, and methods of fuzzification, infer-

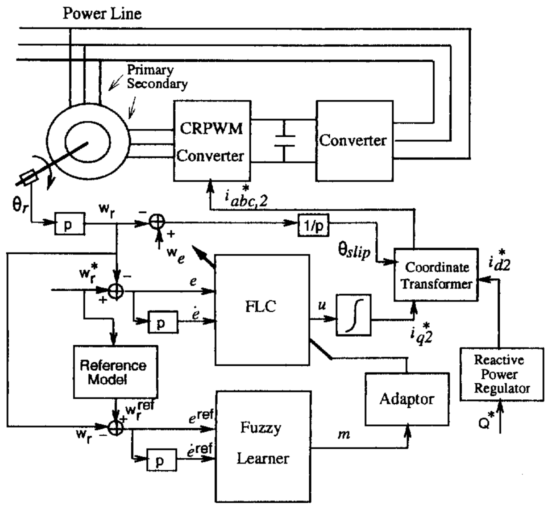

Fig.10 With Adaptive Fuzzy Controller

ences and defuzzification.

Designing of the direct FLC also follows, except that the membership functions for the output fuzzy sets now have triangular shape with fixed width but flexible centers. All of these membership functions are initially centered at zero, representing the fact that the direct FLC initially does not know how to control the machine. These centers are shifted by the fuzzy learner/adaptor such that the output of the direct FLC will control the machine to follow the reference speed response. In each time-step, all of the previously activated fuzzy sets U^j have the centers c^j of their membership functions shifted by the amount of the adapation variable, output of the fuzzy learner m:

$$c^j(t) = c^j(t - dt) + m(t) \tag{13}$$

while the membership functions for the previously unactivated fuzzy sets remain unchanged to have local memory of any previously learned response.

Simulation is conducted for the no-load operation of the system as a variable speed drive to demonstrate both its learning capability and its reference model following capability. Performance specifications are stored in the second-order reference model with the dynamical equation

$$\ddot{\omega}_r^{ref} + K_1\dot{\omega}_r^{ref} + K_2\omega_r^{ref} = K_2\omega_r^* \tag{14}$$

K_1 and K_2 can be calculated from the desired locations of the roots of (14) in the left-hand-site of the imaginary axis. With $K_1 = 1000, K_2 = 250000$ and with an intentional non-optimal design of the FLC normalizing gains, Fig. 11 shows repeated step speed response of the system with the adaptive fuzzy controller. Initially all the membership functions for the output fuzzy sets U^j are centered at zero, while later those for some of U^j related with negative, then zero, then positive, etc., speed errors are automatically positioned in sequence. Note that the desired speed tracking specifications are met through learning, with the solid lines closely follow the dotted lines especially in the later tracking.

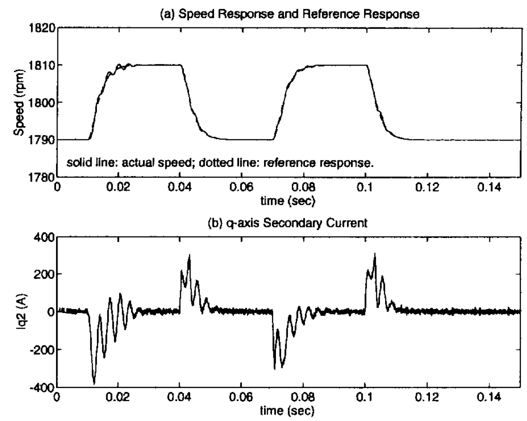

Fig.11 Simulation of Learning Ability

The learned knowledge is stored in the membership functions for the output fuzzy sets, such that later adaptations are faster with less oscillations when a similar situation comes up. However, even though the design of the FLC is no longer critical in long terms, the design of the fuzzy learner posts a bottlenet to the actual performance, as demonstrated by the oscillations in the controller output as it drives the speed to instantaneously track the reference response.

V. Active and Reactive Power Capacities

It is useful to study the active and reactive power capacities of the generating or motoring system, as it is different from other existing variable speed systems. In addition, control of the line side converter need to be explained.

For the squirrel cage induction generator, to maintain the air-gap flux, a reactive current must be provided from the stator circuit, i.e. the generator requires lagging current or reactive power while generating active power. In the new generating system, the secondary circuit of the DEBRM is connected to the static power converter unit with a dc link, which is eventually connected to the main power line. This secondary circuit can play the role of excitation circuit. The power converters and the dc link are important in supporting this excitation circuit.

With the helpful secondary circuit, we can now first show that the primary circuit can send out reactive power, i.e. operates at leading power factor if the reference current direction is as shown in Fig. 12.

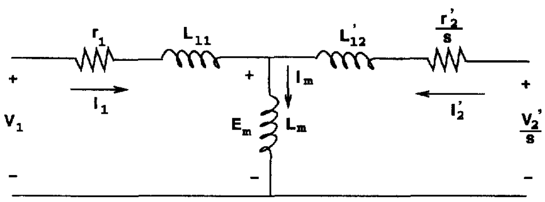

Fig.12 Steady-state Equivalent Circuit

We can assume naturally that $P_1 < 0$. For a leading I_1, the needed secondary current I_2' is found in Fig. 13(a). The rotation speed may be sub-synchronous, when slip power is supplied to the secondary circuit, i.e., $P_2 > 0$; or super-synchronous, when slip power is substracted from the secondary circuit, i.e., $P_2 < 0$. For sub-synchronous and super-synchronous operations, the secondary voltage are shown in Fig. 13(b,c), respectively. In both cases, the secondary circuit obtains lagging current from the power converter, i.e., $Q_2 > 0$. Understandably, it is this reactive current that maintains the air-gap flux. In fact, this current in the secondary circuit can be decomposed into two components, one as air-gap field magnetizing current and the other one to produce a magnetic field to balance that by the primary current, much the same as the role of the stator current in a squirrel cage induction machine.

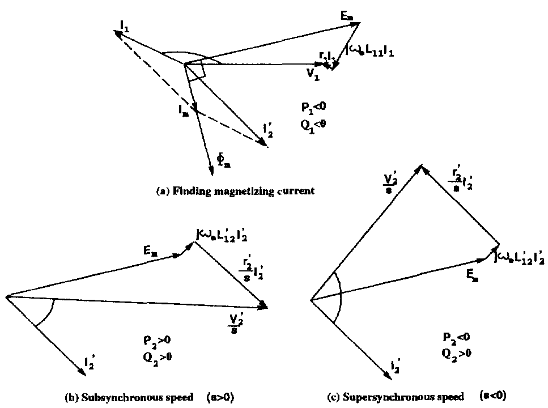

Fig.13 Magnetizing Current and Reactive Power

The power converter provides the secondary circuit with reactive current, whenever reactive power is to be generated in the primary circuit, in both sub- and super-synchronous speed ranges. Furthermore, the direction of the reactive power in the primary circuit can also be reversed, supplying magnetizing current solely or jointly with the secondary circuit.

The above flexibility in primary and secondary reactive power directions is made possible by the availability of secondary circuit reactive current, which is supported by the machine side converter and the capacitive dc link. The line side converter is capable of controlling the direction and amount of the reactive power in its ac side. As a result, the overall power factor of the generating system can be leading, unity or lagging.

Both the converters used are pulse-width-modulated voltage source inverters (PWM VSI), with current regulation at their ac sides. Bi-directional switches or uni-directional switches with freewheeling diodes can be used, enabling the converters to handle bi-directional power flow. This type of connverters is characterized as being able to modulate the ac side current at any power factor, as long as the dc bus voltage is high enough.

If the dc bus voltage is not sufficiently high, loss of control for the converters will occur. The minimum dc bus voltage can be found to be [10]

$$V_{dc,min} = \frac{3\sqrt{6}}{\pi} V_{ac,rms} \tag{15}$$

where V_{ac} is the converter ac side phase voltage. This requirement is met easily since normally the dc link operates at a much higher voltage, maintained a constant as shown next.

The dc link, supported by a capacitor, has the following dynamical equation

$$C\frac{dV_{dc}}{dt} = i_{dc1} - i_{dc2} \tag{16}$$

with dc link currents i_{dc1} and i_{dc2}. For the dc link to maintain a constant voltage at all time after the initial charging of the capacitor, it is necessary that

$$i_{dc1} = i_{dc2} \tag{17}$$

Assume no losses in the converter switches, then the energy balance of the converters gives

$$V_{dc} i_{dc2} = P_2 \tag{18}$$

and

$$V_{dc} i_{dc1} = P_l \tag{19}$$

where P_l is the slip active power at the ac side of the line side converter. Then with (17), it is essential that

$$P_2 = P_l \tag{20}$$

Equ. (20) indicates that to maintain a constant dc link voltage, the slip active power is transferred instantaneously to the main power line, for super-synchronous speed operation, or vice versa for sub-synchronous speed operation. This is made sure by the line side converter current regulation action. Another result of (20) is that the overall generated active power of the system equals the electromagnetic power minus machine losses and converter switching losses, i.e.

$$P = P_e - P_{loss} - P_{sw} \tag{21}$$

At the ac side of the line side converter, while the active power is controlled from (20), the reactive power remains flexible, considering that the current commands include current magnitude and power factor, which can be translated into dq axis currents. Therefore, if the primary circuit reactive power Q_1 has been controlled to minimize the coppor losses, the overall reactive power command Q^* can be balanced by the reactive power Q_l generated by the line side converter, i.e.

$$Q^* = Q_1 + Q_l \tag{22}$$

The limits of such a flexible reactive power control shall now be discussed. Since the reactive powers linking to the machine and the reactive power at the line side converter are not directly related, their limits can be separately assessed.

One of the limits of the reactive power control in the primary circuit of the machine is closely related with a limit of the active power. This limit is set by the primary circuit current handling capability, which is determined by thermal rating of the windings. A circular boundary can be drawn, with the active power and the reactive power as the two axes bounded by MVA rating of the machine.

Much the same as the role of the field current limit in a synchronous generator [11], the secondary circuit current limit sets another limit on the reactive power in the primary circuit. Naturally, the secondary winding thermal limit should not be violated, and magnetic saturation limits the magnitude of the air-gap flux. However, as designed, it is the converter rating that poses the limit, which constrains the secondary winding currents. The converter rating, the lower the better considering its high cost and switching losses, is selected to match the intended speed range. As an example, for the speed range to be within 50% below and beyond synchronous speed, both the converters should be rated at 50% of the machine KVA rating. The effect of the secondary current limit on the primary reactive power limit is demonstrated in Fig. 14, in which for a fixed primary voltage, the secondary current limit sets a limit for the magnetizing current, which sets a limit for the counter emf, setting a limit for the primary current. As shown, the maximum generated primary reactive power is reduced during a certain range of leading power factor, which can be solved for easily. This is due to the fact that for the primary circuit to send out the relatively large amount of reactive power, the secondary circuit is overwhelmed with needed large reactive current, which is prevented by the converter rating.

Similarily, the reactive power from the line side converter is limited by the converter rating. As a result, the overall reactive power is limited, jointly by the winding thermal constraint and the line side converter rating, except at the specific range of leading power factor deter-

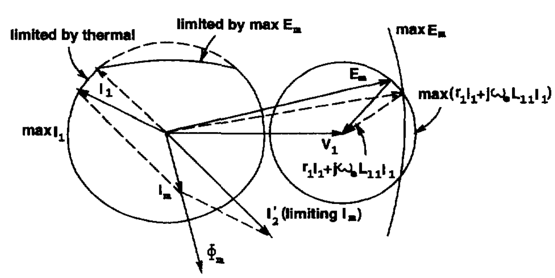

Fig.14 Primary Circuit Active and Reactive Power Capacities (Generator and Motor)

mined in Fig. 14. Fig. 15 shows the overall active and reactive power capacities, where I_l is the ac current of the line side converter and I is the overall current of the system. Note that in both Fig. 14 and Fig. 15, the generated active power is assumed not to be limited by the prime mover. Under variable speed operation, wind turbine optimal power-speed profile is tracked to extract maximum available energy from the wind, therefore active power generated is not intended to meet any command from the bulk power system.

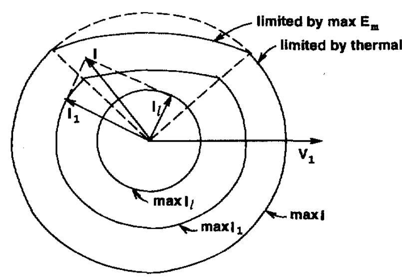

Fig. 15 Overall Active and Reactive Power Capacities

In conclusion: the system can operate at leading, unity or lagging power factor; generally, the overall reactive power is limited by the thermal rating of the windings and the rating of the static power converter, which depends on the intended speed range for operation; for a specific range of leading power factor operation, the overall reactive power becomes limited solely by the converter rating.

VI. Conclusion

In this paper, fuzzy logic control of a high performance variable speed generating system is developed. The doubly excited brushless reluctance machine is the core component of the system. Advantages of the system include: high efficiency, rugged structure with good reliability and low costs, good flexibility and minimum harmonic distortions.

Both the fuzzy logic controllers are of adaptive nature. The direct fuzzy controller has the following features: not dependent on a mathematical model of the machine and the converter; reduced numbers of sensors and observors; no need for PID type regulators; rejection of parameter variations and disturbances. With the adaptive fuzzy controller, the generating system achieves learning and adaptation ability, and its performance is less sensitive to the design of the direct fuzzy logic controller.

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