Identification of Parameters of an A. C. Machine from Standstill Time Domain Data

A. Keyhani S. I. Moon L. Xu

Department of Electrical Engineering
The Ohio State University
Columbus, Ohio 43210

Abstract. This paper presents an evaluation of the performance of the maximum likelihood (ML) method when used to estimate the linear parameters of a synchronous machine model from the standstill time-domain flux decay test data. It is shown that a unique set of parameters can be obtained and the noise effects can be dealt with effectively when the ML estimation technique is used. The results of study also show that accurate machine parameters can be identified even when signal-to-noise ratio is as low as 200:1.

Introduction

Many papers have been published [1-11] on synchronous machine modeling using frequency and time-domain parameter estimation techniques. However, the effects of noise on synchronous machine parameter estimation have not been studied extensively [11,12]. In reference [12] a quasilinearization-based least-square algorithm was used to estimate machine parameters from noise-corrupted data. The percent error of estimated parameters was as high as 32% for noise-free data and 259% for noise-corrupted data. In reference [11], the generalized least-square technique was employed, and the percent error in some of the estimated parameters was as high as 100%. These studies confirm that the least-square algorithm is very sensitive to noise-corrupted data .

In our approach, we have considered four issues: 1) testing procedures, 2) model structural identification, 3) estimation techniques, and 4) initialization requirements. In the work reported in the reference [16], the testing procedure was limited to the standstill frequency response test (SSFR). In our previous work [1,2], first we used synthetically generated and noise-corrupted SSFR data to develop and validate our estimation procedure.

The structural identification problem needs to be studied from the electromagnetic analysis of the generator using the finite-element method [5]. Through the studies of the flux distribution within the machine, the corresponding magnetic equivalent circuits can be determined. From such studies, it has suggested [9,10] that a generator may be represented by the standard, SSFR2 or SSFR3 equivalent circuits. However, other equivalent circuits should also be considered since the model structural identification is at best an approximate representation of the physical process under consideration. We believe that since the number of models that can be considered is limited, the performance of all candidate models should be evaluated.

From a theoretical view point, the Akaike Final Prediction Error (AFPE) is best suited for discriminating between two sets of models. However, we feel that this test is not necessary, since most modern power system stability programs can only use the standard, SSFR2 and SSFR3 models. Furthermore, the selction of a suitable model is based on its performance as compared with measured responses of power system disturbances. For such a comparison, the machine models require adequate representation of saturation effects.

The robustness of the parameter estimation technique cannot be studied from the analysis of measured responses. If one assumes a model structure and then proceeds with estimating its parameters from actual measurements, then the structural error and the effect of noise in the measurements will result in inaccurate parameters. It will not be clear whether the discrepancy between the simulated model response and the measured response is due to the effect of noise on the parameters, inadequacy of the assumed model structure, or both. Therefore, the structural identification problem and the parameter estimation problem should be studied separately.

For example, the parameters of the SSFR3 model can be estimated from the simulated noise-corrupted data using the following system equation:

$$f_i(\bar{x}) = \bar{y}_i + g_i(\bar{x}, \bar{y}) + \xi_i = 0 \qquad (1)$$

where i=1, ... , 8. The $\bar{y}$ is a known vector, and it is given in terms of the estimated time constants and the gain of $L_d(s)$ and $sG(s)$ [1]. The $\bar{x}$ is an unknown vector which represents the seven parameters of the d-axis circuit model. These equations are nonlinear in nature and are not consistent with each other. This is due to the noise ξ_i imbedded in vector $\bar{y}$. Naturally, these equations would be consistent if simulated noise-free data are used in the analysis. A unique solution will be obtained regardless of the equation which is discarded in the solution process. However, when the measured data are used, the equations are inconsistent because of the inherent noise in the data, and multiple solutions are obtained depending on which equation is ignored in the solution process. We have studied these problems in our previous work [1,2].

One may suggest that multiple solutions for the machine parameters are "equally acceptable." We believe multiple solutions are due to the effect of noise on estimation method, the local convergence of the nonlinear optimization problem, and the error in the assumed model structure. There is only one set of physically realizable machine parameters which will result in the global convergence of the nonlinear optimization problem's cost function. The "equally acceptable solution" may also mean that the measured responses match the simulated responses. The estimated parameters do not represent the physical parameters of the generator. They are due to the local convergence of the optimization problem's cost function. One should be cautious when using such parameter values in power system stability studies.

In the present study, we have assumed that the turns ratio between the armature winding and the field winding is known. Then a simple d.c. flux decay test is used to collect the input and output data of the machine for estimation. This off-load test method has advantage over the on-line test methods because of the expense and difficulty of obtaining transient test data from loaded machines. The d.c. decay test [3-4] can be performed at the factory or at the power plant, with machine stationary. This test requires only a supply of direct current.

Problem Description

The fidelity of synchronous machine models is affected by the proposed model structures, the quality of the experimental data used to identify the model's parameters, and the robustness of the estimation technique. One question needs to be answered. – If the assumed model structure is correct, then can one obtain a unique estimate of the parameters from noise-corrupted transient response data? The answer to this question cannot be found from measurements, since the measurements are made on a machine with a complex, high order rotor circuit, with unknown structure and unknown parameters.

If one assumes a model structure and then proceeds with estimating its parameters from actual measurements, then the structural error and the effect of noise in the measurements will result in inaccurate parameters. Therefore, it will not be clear whether the discrepancy between the simulated model response and the measured response is due to the effect of noise on the parameters, inadequacy of the assumed model structure, or both. Therefore, the structural identification problem and the parameter estimation problems should be studied separately. There is a need to show that the measurement noise will not corrupt the estimated parameters when the parameters of an assumed structure are estimated from the input/output measurements.

To study this problem, a machine model with a known parameter set is simulated and then the data are noise-corrupted using known noise distribution. The objective is to estimate the parameters of the machine model from the noise-corrupted data and evaluate the effect of noise on the estimated parameters by comparison with the known parameters. The problems studied can be stated as:

- Using the d.c. flux decay data, how large can the noise level be set such that no significant disagreements between the estimated and the actual parameters exist?
- How close should the initial parameters be selected in the estimation to guarantee the algorithm convergence to the correct estimates?

Standstill Synchronous Machine Model for Time-Domain Parameter Estimation

When the standstill measurements are used for estimation, the mathematical formulation for the machine can be broken down into two separated state-space representations for d- and q-axis respectively. This is due to the fact that the coupling speed voltages are equal to zero under the standstill condition. Therefore, the state-space representations of the machine models can be written as following:

For the d-axis model

$$X_d(k+1) = A_d(\theta_d) \cdot X_d(k) + B_d(\theta_d) \cdot U_d(k) + w(k) \quad (2)$$
$$Y_d(k) = C_d \cdot X_d(k) + v(k) \quad (3)$$

For the q-axis model

$$X_q(k+1) = A_q(\theta_q) \cdot X_q(k) + B_q(\theta_q) \cdot U_q(k) + w(k) \quad (4)$$
$$Y_q(k) = C_q \cdot X_q(k) + v(k) \quad (5)$$

In Eq. (2) and Eq. (4), the definitions of the state variables, output are

$$X_d = [i_d\, i_{1d}\, i_{fd}]^T \,; \qquad X_q = [i_q\, i_{1q}\, i_{2q}]^T$$
$$U_d = [v_d] \,; \qquad U_q = [v_q]$$
$$Y_d = [i_d\, i_{fd}]^T \,; \qquad Y_q = [i_q]$$

The unknown parameter sets for the d- and q-axis are

$$\theta_d = [R_a\, R_{1d}\, R_{fd}\, L_\ell\, L_{ad}\, L_{1d}\, L_{fd}]^T$$
$$\theta_q = [R_a\, R_{1q}\, R_{2q}\, L_\ell\, L_{aq}\, L_{1q}\, L_{2q}]^T \quad (6)$$

In Eq. (2), the input control variable U_d is determined by the d-axis stator voltage alone because, under the standstill test conditions, the field circuit is shorted. In both Eq. (2) and Eq. (4), the quantity w represents the process noise, and the quantity v indicates the measurement noise. In the actual system, the process noise, w is imbedded in the input control variables U_d and U_q, respectively.

The computation of $A_d(\theta_d)$, $B_d(\theta_d)$, $A_q(\theta_q)$, and $B_q(\theta_q)$ from continuous time-domain representation is described in [13]. The explicit parameterization of continuous system representation in terms of θ_d and θ_q is given in reference [2].

Effect of Noise on the Process and the Measurement

The model, which mathematically describes the process, is subjected to the deterministic input at each time instant k. Nature also subjects the process to a random input sequence $\omega(\cdot)$. The sequence $\omega(\cdot)$ is designated as the process noise sequence. It is assumed to be Gaussian with zero mean and covariance matrix $Q(\cdot)$. The covariance matrix Q gives a measure of the intensity of the process noise on the model. A high value of the covairance matrix Q corresponds to a noisy process. The reason for introducing the measurement noise sequence $v(\cdot)$ is that in physical problems, the measurements are inherently subjected to errors. The signal conditioning equipment and sensors introduce measurement noise, which is random. The measurement errors $v(\cdot)$ are assumed to be independent and Gaussian with zero mean value and a known covariance matrix R_0.

Let us denote the variances of $v_{id}(\cdot)$ and $v_{ifd}(\cdot)$ by σ^2_{vid} and σ^2_{vifd}. Also let $v_{id}(\cdot)$ and $v_{ifd}(\cdot)$ represent the measurement noise of i_d and i_{fd}. The assumption that $v_{id}(\cdot)$ and $v_{ifd}(\cdot)$ are independent ensures that measurement of i_d will not introduce additional uncertainty (i.e., measurement noise) in the measurement of i_{fd}. This assumption is not completely true. For example, the use of shunt resistances for current measurements will introduce its own uncertainty in the process variables to be measured. In this paper, however, it is assumed that measurement errors are independent; therefore the covariance $R_0(\cdot)$ is a diagonal matrix, and the diagonal elements represent the variances of the measurement errors. Note that the standard deviations of measurement errors represent the percent errors associated with the sensors. The accuracy of the sensors may be known from the "manufacturer data" or from carefully controlled experiments on the sensors themselves.

The initial covariance R_0 is constructed from the knowledge of sensor errors, and it represents a measure of the prior confidence in the sensors to produce accurate measurements. Strictly speaking, two experiments performed on the same process will not result in identical measurements. Therefore, the covariance of the estimation error is calculated as part of Kalman filter [14,15] for estimating the machine states and then the parameters. The covariance of the estimation error is defined as

$$
\begin{aligned}
R(k) &= COV\,(e(k),\, e(k)) \\
e(k) &= Y(k) - \hat{Y}(k)
\end{aligned}
\tag{7}
$$

In Eq. (7), $Y(k)$ and $\hat{Y}(k)$ represent the measured and the estimated output respectively.

Study Process

The purpose of the study process is to develop a methodology for synchronous machine parameter estimation from the noise-corrupted data due to d.c. flux decay test. For this purpose, synthetic time-domain standstill transient response data were generated using standard synchronous machine model parameters. Then, we will assume the model structure (that is, the number of differential equations representing the machine) and input/output response data are known; however, the model parameters are not known. The objective of the study is to develop a methodology for machine parameter estimation.

For the study process, the synthetic time-domain data are generated using the following procedure. A constant d.c. voltage signal is first applied to the machine model for a specified period of time so that the initial transient responses no longer exist. Then, the d.c. supplied voltage is suddenly removed from the system, and the transient responses are generated for the estimation. In the first stage of the simulation, the noise sequences are not introduced to the signals. Then, in the second stage of the simulation process, the developed time responses were corrupted with Gaussian distributed noise of zero mean and varying degrees of variances depending on the selected signal-to-noise ratios. A low level of signal-to-noise ratio corresponds to a noisy measurement.

In order to simulate the real conditions, the noise must be injected only to those variables which are physically accessible during the machine standstill test. For the d-axis test, the machine rotor is turned to the position where the q-axis coincides with phase a. A diagram depicting the test setup for the d-axis measurements is shown in Fig. 1. In Fig. 1, the adjustable resistor R is used to regulate the inrush current during the startup. A similar test arrangement for q-axis is shown in Fig. 2. In this case, the rotor d-axis is aligned with phase a as shown in Fig. 2.

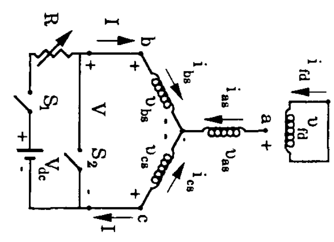

Fig. 1 Rotor position alignment for the d-axis flux decay test.

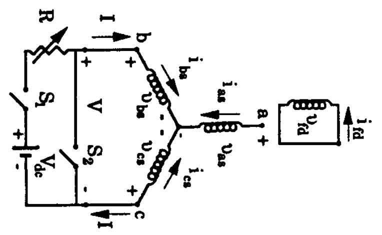

Fig. 2 Rotor position alignment for the q-axis flux decay test.

According to Fig. 1 and Fig. 2, the noise corruption process for the simulated data must be performed on the terminal variables such as i, V and i_{fd}, namely

$$\begin{aligned} \bar{v}(k) &= v(k) + \alpha_1(k) \cdot \omega_1(k) \quad (V) \\ \bar{i}(k) &= i(k) + \alpha_2(k) \cdot \omega_2(k) \quad (A) \\ \bar{i}_{fd}(k) &= i_{fd}(k) + \alpha_3(k) \cdot \omega_3(k) \quad (A) \end{aligned} \tag{8}$$

In the above equation, $\bar{v}(\cdot)$, $\bar{i}(\cdot)$, and $\bar{i}_{fd}(\cdot)$ represent the noise corrupted terminal quantities, and $v(\cdot)$, $i(\cdot)$, and $i_{fd}(\cdot)$ correspond to the noise-free terminal values. The terms, $\alpha_1(k)$, $\alpha_2(k)$, and $\alpha_3(k)$, are defined as

$$\alpha_i(k) = \left| \frac{\text{signal } (k)}{S/N \text{ ratio}} \right| \tag{9}$$

where signal (k) corresponds to the noise-free signal, and S/N ratio indicates the signal-to-noise ratio used in the noise corruption process. The S/N ratio is defined as

$$S/N \text{ ratio } = \left[\frac{\sum_{k=0}^{N} \text{signal}^2 \, (k)}{\sum_{k=0}^{N} (\alpha_i(k)\omega_i(k))^2} \right] \tag{10}$$

The sequence $\omega_i(\cdot)$ in Eq. (10) is Gaussian distributed with zero mean and unity variance.

In Eq. (8), the noise-free variables $v(k)$, $i(k)$, and $i_{fd}(k)$ can be calculated based on the noise-free simulated stator and field quantities in dqs reference frame using Park's transformation and the specified rotor positions in Fig. 1 and Fig. 2, namely

$$\begin{aligned} v(k) &= -\sqrt{3}\, v_d(k) \\ i(k) &= -\frac{\sqrt{3}}{2}\, i_d(k) \end{aligned} \tag{11}$$

for the d-axis flux decay test, and

$$\begin{aligned} v(k) &= \sqrt{3}\, v_q(k) \\ i(k) &= \frac{\sqrt{3}}{2}\, i_q(k) \end{aligned} \tag{12}$$

for the q-axis flux decay test. Once the data are corrupted with additive noise, the resultant d- and q-axis stator quantities can be calculated using Eq. (11) and Eq. (12) again. However, $\{v(\cdot),\ i(\cdot)\}$ must be replaced by $\{\bar{v}(\cdot),\ \bar{i}(\cdot)\}$.

Maximum Likelihood Parameter Estimation

To identify the machine parameters, the maximum likelihood estimation algorithm (ML) is used. (See Fig. 3.) The likelihood function used is defined as [14]

$$L(\theta) = \prod_{k=1}^{N} \left[\frac{1}{\sqrt{(2\pi)^m \det(R(k))}} \exp\left(-\frac{1}{2}\, e^T(k) R^{-1}(k) e(k) \right) \right] \tag{13}$$

where $e(\cdot)$, $R(\cdot)$, N, and m denote the output estimation error, the covariance of the output estimation error, the number of data points, and the dimension of Y, respectively. The output estimation error, $e(\cdot)$, and the corresponding covariance, $R(\cdot)$, are defined by Eq. (7). The estimation steps are given in reference [2].

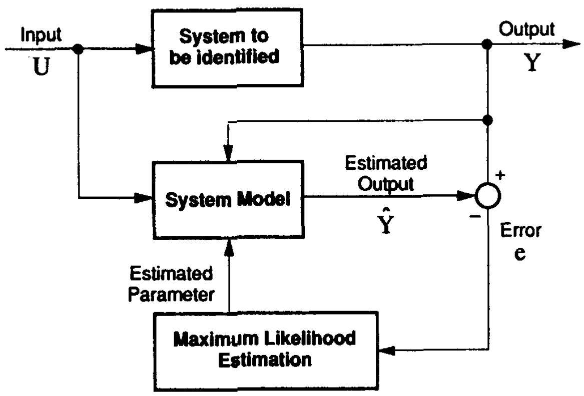

Fig. 3 Maximum likelihood estimation.

Estimation Procedure

According to the above discussion, the entire estimation effort involves two phases. Phase one is to obtain adequate flux decay test information, and Phase two is to obtain the machine model parameter estimates using maximum likelihood method. For the benefit of the future discussion, the estimation procedure to establish the standard model parameters is summarized below as:

1. Perform the standstill time-domain flux decay test with the rotor in both positions of the d-axis and the q-axis.
2. Transform the available measurement data to dqs reference frame using Park's transformation, in particular, using Eq. (11) for d-axis and Eq. (12) for q-axis.
3. Estimate both the d- and q-axis standstill model parameters using the available test data calculated in step 2 and the maximum likelihood estimation algorithm.

Results

The q-axis flux decay test responses are shown in Fig. 4. The initial stator currents prior to the closing of switch S_2 (see Fig. 1 and Fig. 2) are 150 A for both d-axis and q-axis. The S/N ratio used in Fig. 4 is 200:1. Three other different noise levels are also used in the estimation study to see the effect of noise on the outcome of the estimation. The sampling interval used in the simulation is 0.001 second, and for each signal 1800 data points are generated. For the flux decay test, the voltage across the machine terminals is zero immediately after the closing of switch S_2. Henceforth, the terminal voltage is not plotted in Fig. 4. However, prior to the flux decay test, the d.c. terminal voltage and current have already reached steady-state stage. Consequently, using Ohm's law, the armature winding resistance R_a can be easily computed before the startup of the flux decay test. The computed R_a, then, can be used in the process of estimating other machine elements under the flux decay responses.

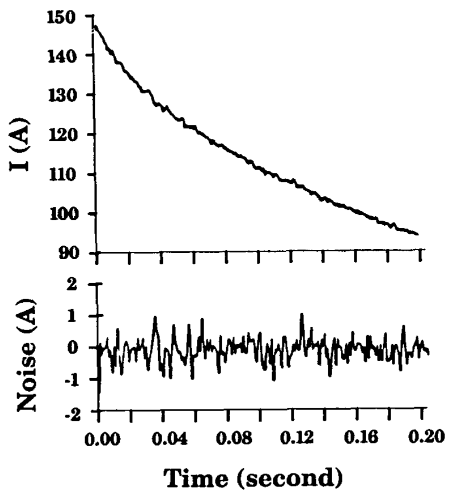

Fig. 4 Synthetic test data and added noises for q-axis D.C. flux decay test with initial stator current of 150 A.

The results of the estimation based on the responses in Fig. 4 are listed in Table 1 and Table 2. The percentage errors listed in these tables are calculated according to the following formula

$$\%\ \text{error} = \frac{(\text{Estimated Value}) - (\text{Original Value})}{(\text{Original Value})} \times 100\%$$

In both d- and q-axis estimation, the unknown initial parameters are selected at 20% of the actual values. The armature resistance R_a is computed using the steady-state current prior to the flux decay test. Two hundred points are used for such purpose. For the estimation of d-axis parameters, R_a is set to the specified values, and L_l is assumed to be one of the unknown parameters which needs to be estimated. For the estimation of q-axis unknowns, R_a and L_l are set to the values obtained in the steady-state test and the d-axis parameter estimation.

As shown in Table 1, the d-axis parameters can be estimated quite accurately under all noise levels considered. For the case of q-axis model estimation, the second q-axis damper winding inductance L_{2q} can not be estimated accurately when the S/N ratio is at 200:1 (see Table 2). In order to keep the q-axis estimated parameters to be within 95% of the actual values, the lowest S/N ratio for the q-axis flux decay test may have to be limited above 500:1.

Machine Parameters in p.u.	Original Value in p.u.	Estimated Parameter Values in p.u.							
		S/N Ratio I, I_{fd} - 2000:1		S/N Ratio I, I_{fd} - 1000:1		S/N Ratio I, I_{fd} - 500:1		S/N Ratio I, I_{fd} - 200:1	
		Est. Value	% Error	Est. Value	% Error	Est. Value	% Error	Est. Value	% Error
R_a	0.00411	(0.00410)	-0.24	(0.00410)	-0.24	(0.00410)	-0.24	(0.00409)	-0.49
R_{1d}	0.01865	0.01859	-0.38	0.01858	-0.38	0.01855	-0.54	0.01847	-0.97
R_{fd}	0.00105	0.00105	0.00	0.00105	0.00	0.00105	0.00	0.00105	0.00
L_l	0.19000	0.18933	-0.24	0.18950	-0.26	0.18961	-0.21	0.18971	-0.15
L_{ad}	1.73000	1.72624	-0.22	1.72578	-0.24	1.72553	-0.26	0.72714	-0.17
L_{1d}	0.13050	0.13067	0.13	0.13130	0.61	0.13206	1.20	0.13484	3.33
L_{fd}	0.16380	0.16340	-0.24	0.16346	-0.21	0.16337	-0.26	0.16331	-0.30

Table 1 Estimated d-axis parameter values from 150 A initial current flux decay test data.

Machine Parameters in p.u.	Original Value in p.u.	Estimated Parameter Values in p.u.							
		S/N Ratio I - 2000:1		S/N Ratio I - 1000:1		S/N Ratio I - 500:1		S/N Ratio I - 200:1	
		Est. Value	% Error	Est. Value	% Error	Est. Value	% Error	Est. Value	% Error
R_a	0.00411	(0.00410)	-0.24	(0.00410)	-0.24	(0.00410)	-0.24	(0.00409)	-0.49
R_{1q}	0.00585	0.00583	-0.34	0.00584	-0.17	0.00584	-0.17	0.00585	0.00
R_{2q}	0.02475	0.02470	-0.20	0.02472	-0.12	0.02476	0.04	0.02487	0.48
L_l	0.19000	(0.18955)	-0.24	(0.18950)	-0.26	(0.18961)	-0.21	(0.18971)	-0.15
L_{aq}	1.66000	1.65556	-0.27	1.65522	-0.29	1.65452	-0.33	1.65248	-0.45
L_{1q}	0.54450	0.54278	0.32	0.54277	0.32	0.54277	-0.32	0.54282	-0.31
L_{2q}	0.08441	0.08520	-0.94	0.08615	2.06	0.08807	4.34	0.09405	11.42

Table 2 Estimated q-axis parameter values from 150 A initial current flux decay test data.

Conclusion

In this paper, the circuit elements of the standard synchronous machine model are estimated using the time-domain flux decay test data. The results of the estimation show that this type of standstill test data excites the machine dynamic modes and can be used to estimate the machine parameters provided that the signal-to-noise levels in various measured signals are kept within a range of 200:1. If the signal-to-noise ratio is reduced below the value of 500:1, the

q-axis damper winding parameter L_{2q} may also be estimated highly accurately.

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Appendix A

Nomenclature

$\hat{(\cdot)}$	:	Estimate of $(\cdot)$
$[\ ]^T$	:	Transpose of $[\ ]$
$E[\cdot]$	:	The operation of taking the expected value of $[\cdot]$
$E[x]$	:	Average value of the sequence $X(\cdot)$ designated by $\bar{X}$
$COV(X)$	:	$E[(X-\bar{X})(X-\bar{X})^T]$
$\omega(\cdot)$	:	Process noise sequence
$v(\cdot)$	:	Measurement noise sequence
$X(\cdot)$	:	State vector
$Y(\cdot)$	:	Measured output vector in the presence of noise
Q	:	Covariance of the process noise sequence
R_0	:	Initial covariance of the measurement noise sequence
$R_1(\cdot)$	:	Covariance of the process noise
$R_2(\cdot)$	:	Covariance of the estimation error
$e(\cdot)$	:	Estimation error, $e(k) = Y(k) - \hat{Y}(k)$
exp	:	The exponential operator
det	:	Determinant
$\hat{Y}(k\|k-1)$	:	The estimated value of $Y(k)$ at time instant k given the data up to $k-1$
$U(\cdot)$	:	Input function
$\theta(\cdot)$	:	Parameter vector