achieved by the time variable transformation can be equivalently expressed by writing \( k(\theta) = k(x, y, z, \theta) \), giving

\[
\nabla^2 \theta + \frac{g}{k_0} = \frac{1}{k(x, y, z, \theta)} \frac{\partial \theta}{\partial \theta}
\]

(3)

This equation is now fully linear, but \( k(x, y, z, \theta) \) is not known a priori and must be obtained self-consistently, as described in [3].

The time variable transformation then corresponds essentially to the replacement, \( k(\theta) \rightarrow k(\theta) = k_\theta \), in the numerical solution of the nonlinear problem. \( k_\theta \) equals \( k(x, y, z, \theta) \) at spatial node \( i \) and at a given time step.

Such a solution can be implemented within the authors' fully analytical thermal resistance matrix approach. Writing

\[
\nabla^2 \theta + \frac{g}{k_S} = \sum_j f_j(x, y, z) \alpha_j(t)
\]

(4)

\[
\frac{1}{k(\theta)} \frac{\partial \theta}{\partial \theta} = \sum_j f_j(x, y, z) \alpha_j(t)
\]

(5)

where the \( f_j \) are specified and the \( \alpha_j \) are to be determined, eqn. 4 can be solved analytically in rectangular subvolumes, as described by the authors in [5]. Evaluating this analytical solution at points, \( (x_0, y_0, z_0) \), temperature rises above some boundary condition defined constants are calculated as

\[
\Delta \theta_i(t) = \sum_j R_{ij} \alpha_j(t)
\]

(6)

The time-independent matrix, \( R \), is not the thermal resistance matrix in the conventional sense, but is constructed using the same accelerated series methods [6]. \( R \) for complex structures is readily obtained by combining rectangular subvolume solutions [5].

Substituting for the \( \alpha_i(t) \) from eqn. 6, in eqn. 5, a system of coupled first order ordinary differential equations for the \( \Delta \theta_i(t) \) is obtained. Choosing the simplest explicit, numerical, time domain solution scheme for illustration, the recursive algebraic relation

\[
\Delta \theta_i^{(n+1)} = \sum_j M_{ij}^{(n)} \Delta \theta_j^{(n)}
\]

(7)

is obtained at timestep \( n + 1 \), where system matrix \( M_i^{(n)} = \delta_{ij} + \sum_{S} \delta \theta_i^{(n)} f_j f_{S} R_{ij}^{(n)} \) and \( f_j = f_j(x, y, z) \). With proper choice of the \( f_j \) and placement of the \( \alpha_i \), the number of nodes required in this dense matrix formulation, \( j = 1, ..., N \), need be no more than that of a conventional boundary element method [3].

This approach is shown for the example presented by Krabbenhoft and Damkilde in [1]. Using, \( f_j(x) = |x - y| \), and imposing the boundary conditions, \( \delta x = 0 \), \( \theta = \theta_0 \) and \( \delta x = L, \theta = 0 \), of Fig. 1 is obtained. Curve (ii), solid line, shows the fully analytical Fujita solution for \( \lambda = 0.965 \); curve (i), dashed line, shows the result of the extended transformation approach. Agreement is essentially exact at all but the smallest times.

**Conclusion:** The time variable transformation described in [2], for solution of the nonlinear diffusion problem, is demonstrated to provide an alternative approximation to use of a mean diffusion, for moderate nonlinearities such as those occurring in semiconductors. Importantly, this approach is compatible with compact thermal model construction, for rapid, coupled electro-thermal CAD. It avoids the need for computationally intensive numerical thermal solutions which are totally impractical in the necessarily iterative treatment of large, coupled electrical-thermal problems.

For an arbitrary temperature dependence of material parameters, the method described in [2] can be further developed within the authors' fully analytical thermal resistance matrix framework providing an alternative to conventional numerical approaches, such as finite difference, finite element or boundary element methods. However, this solution is not as economical as the original approximation. The problem of the minimal compact description of strongly temperature dependent diffusivity therefore still requires solution. Further development of a time variable transformation approach may yet prove useful in this context.

---

**References**


ICP-RIE. These 0.25μm gate-length devices exhibited gate to drain breakdown over -90 V, leakage current as low as 4μA at $V_{DS} = -25$ V, $f_T$ of 48 GHz and $f_{max}$ of 108 GHz. These results demonstrate significant improvements for recessed-gate AlGaN/GaN HEMTs.

The layer used in the present study was grown on silicon carbide substrates by metalorganic chemical vapour deposition (MOCVD). The epitlayer consists of 2 μm undoped GaN, 3 nm undoped Al$_{0.25}$Ga$_{0.75}$N spacer, 10 nm Al$_{0.33}$Ga$_{0.67}$N with a doping level of $6 \times 10^{19}$ cm$^{-3}$, 10 nm undoped Al$_{0.25}$Ga$_{0.75}$N and 40 nm thick Si-doped (9$\times$10$^{18}$ cm$^{-3}$) n-GaN cap layer. Hall measurements gave a sheet carrier concentration of $1.35 \times 10^{13}$ cm$^{-2}$ and an electron mobility of 75 cm$^2$/Ns.

Fig. 1. DC $I_{DS}$-$V_{DS}$ characteristics of recessed gate 0.25 μm×100 μm AlGaN/GaN HEMTs on SiC substrates

Gate bias was swept from 2 to -8 V in steps of -2 V

- --- as-etched
- - - post-etch annealed

Fig. 2. DC transfer characteristics of recessed gate 0.25 μm×100 μm AlGaN/GaN HEMTs on SiC substrates

Drain bias was 7 V

- --- as-etched
- - - post-etch annealed

Measurements were conducted on both as-etched and post-etch annealed devices using an HP4145B semiconductor parameter analyser. Shown in Fig. 1 are the typical drain current-voltage ($I_{DS}$-$V_{DS}$) characteristics of the devices. The gate was biased from 2 to -8 V in steps of -2 V. The as-etched devices exhibited a maximum drain current density of 730 mA/mm while a slightly higher current density of 770 mA/mm was measured on post-etch annealed devices. Also, pinch-off voltage is lower for the annealed devices. The DC transfer characteristics are shown in Fig. 2. The drain was biased at 7 V. A peak extrinsic transconductance ($g_m$) of 139 mS/mm was measured for the post-etch annealed devices while as-etched devices produced a peak $g_m$ of 109 mS/mm. The effect of post-etch anneal on the gate to drain I-V characteristics is shown in Fig. 3. The gate-to-drain breakdown voltage increased from -27 V to over -90 V after post-etch anneal. Also, gate leakage current was significantly reduced and is as low as 4 μA at $V_{DS} = -25$ V. These improvements in device performance are attributed to the recovery of plasma damage by annealing. To the best of the authors' knowledge, this is highest ever reported value of gate-drain breakdown voltage for recessed-gate GaN based HEMTs.
low gate leakage current. The measured values of gate-drain breakdown voltage of over 90V, \( f_r \) of 48GHz and \( f_m \) of 108GHz are the highest ever reported data for recessed-gate AlGaN/GaN HEMTs.

Acknowledgments: This work was supported at UIUC by ONR under contract no. N00014-01-1-0001, Air Force under contract no. AF956-406029, and Triquint Corporation.

© IEE 2001
Electronics Letters Online No: 20010999
DOI: 10.1049/el:20010999

V. Kumar, W. Lu, F.A. Khan, R. Schwindt and I. Adesida
(Department of Electrical and Computer Engineering and Microelectronics Laboratory, University of Illinois at Urbana
Champaign, IL 61801, USA)

E. Piner (ATM/Epitrronics, Phoenix, AZ 85027, USA)

References

Threshold analysis in wavelet-based denoising
L. Zhang, P. Bao and Q. Pan

The hard threshold \( t = \sigma \) is efficient in wavelet-threshold-based nonlinear filtering. In general, the optimal constant \( c \) would vary with the signal and the added noise. The nearly optimal choice of \( c \) by minimising \( R(c) \) is equivalent to the mean square error (MSE) of the recovered signal, is discussed. Experiment shows that \( R(c) \) is consistent with the MSE.

Introduction: In wavelet-based signal denoising, an intuitive and efficient approach is to apply the preset threshold to the wavelet coefficients. Donoho [1] first gave a soft threshold \( t = \sigma \sqrt{2 \log M} \), where \( \sigma \) is the standard deviation of noise and \( M \) is the length of signal. While this threshold possesses some minimax properties, it is non-intuitive and varies with \( M \) and the added noise. In this Letter, a function \( R(c) \) that approximates an equivalent to the mean square error (MSE) of the recovered signal is constructed. The nearly optimal \( c \) can be determined by minimising \( R(c) \).

Hard threshold-based denoising by wavelet transform: Suppose there is a sequence of observations \( y_i = x_i + n_i \), \( i = 1, 2, \ldots, M \), where \( n_i \sim N(0, \sigma^2) \) is white Gaussian noise. The goal is to estimate signal \( X \) from \( Y \). Donoho [1] first developed a wavelet shrinkage method by a soft threshold \( t = \sigma \sqrt{2 \log M} \). Some impressive results were reported based on hard thresholding [2]. The procedure can be described as follows. First the sequence \( Y \) is transformed into wavelet coefficient \( W \); then hard threshold \( t = \sigma \) is applied on \( W \):

\[
W_i = \begin{cases} 
W_i & |W_i| \geq t \\
0 & |W_i| < t 
\end{cases}
\]

where \( c \in (3, 4) \) is a constant; finally the estimation \( \hat{Y} \) is reconstructed from \( W \).

In general, for different signals and noise, the optimal \( c \) will be different. We now construct a function \( R(c) \), nearly equivalent to the MSE of \( \hat{Y} \), to determine \( c \).

Determination of \( c \): Orthogonal wavelet transform (OWT) is linear. Thus we will have \( W_Y = W_X + W_N \), where \( W_Y \), \( W_X \) and \( W_N \) denote the OWT of observation \( Y \), signal \( X \) and noise \( N \), respectively. Similarly, we have \( W_Y = W_X + W_N \), where:

\[
\begin{align*}
W_X(i) &= W_X(i), \quad W_N(i) = W_N(i) \quad |W_N(i)| \geq t \\
W_N(i) &= 0, \quad W_N(i) = 0 \quad |W_N(i)| < t
\end{align*}
\]

We also have \( \hat{Y} = \hat{X} + \hat{N} \), where \( \hat{Y}, \hat{X}, \hat{N} \) are the inverse OWT of \( W_Y, W_X, W_N \).

Obviously, the optimal \( c \), i.e. \( t \), should minimise the MSE of \( \hat{Y} \):

\[
E[\hat{Y}^2] = E[(\hat{X} - X)^2] = \min
\]

Because \( E[\hat{Y}^2] \) is independent of \( c \), it is equivalent to minimising the following function:

\[
\text{Error}(c) = E[\hat{Y}^2] = E[X^2]
\]

Since OWT is orthonormal, from eqn. 2 and eqn. 3, \( E[\hat{W}_X^2] = E[X^2] \), we get:

\[
\text{Error}(c) = E[\hat{W}_X^2] = E[\hat{W}_N^2] = E[\hat{W}_N^2]
\]

Suppose \( c \) points will be eliminated in \( \hat{W}_Y \), we have:

\[
E[\hat{W}_Y^2] = \frac{1}{M} \sum_{i=1}^{M} W_X^2(i) = \frac{1}{M} \sum_{j=1}^{K} W_N^2(j)
\]

where \( W_N^2 \) denotes the \( K \) eliminated points in \( W_N \).

It is almost true that if \( W_N(i) < t \) then \( \hat{W}_N(i) < t \). This statement is validated by the tests on two typical signals in Fig. 1. Let \( t = \sigma \) and \( c \) increase from 1 to 4 with step-length 0.3. Denote \( K \) the number of points satisfying \( W_N(i) < t \) and \( K_\sigma \) the number of points satisfying both \( W_N(i) < t \) and \( W_X(i) < t \). The averaged results of \( K_\sigma/K \) generated by the Monte Carlo experiment in signal-to-noise ratio (SNR) are listed in Table 1. In fact, when \( t > \sigma \), for noise in any scale, \( K_\sigma/K \) is nearly equal to 1, which implies that when \( W_X(i) < t \), \( W_N(i) < t \) holds with high probability. The wavelets used in the experiments are Haar (for Blocks) and four taps orthogonal wavelet (for Bumps).

![Fig. 1 Two typical signals and their noisy versions](image)